

Secrets  
of the  
Mathematical  
Contest  
in  
Modeling

v. 1.8, 1/09, K. S. Cline

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## Introduction

The following is my own personal strategy of how to attack the MCM. It is certainly not the only way, and I make no claims that it is the best way. Participating in the MCM was the most fun, challenging, exciting, frustrating, and exhausting thing I ever did as an undergraduate. I participated three years: The first time, I was a barely out of Calculus, and the MCM hit me like a freight train. I was totally unprepared, utterly blindsided, and my team ended up with a Successful Participant ranking. I spent hours analyzing what went wrong, and plotting my revenge. The second year, we did better, but I still didn't have a handle on things, and we didn't know where to attack, ending up with an Honorable Mention. By the third year ('98), I was out for blood. I went over the previous years problems, studying the anatomy of a good paper, relentlessly preparing for the intellectual battle of a lifetime. This time my team clicked, and we annihilated the problem – receiving an Outstanding, the highest possible ranking. In the years since I graduated, I have coached teams both at the University of Colorado at Boulder 2000 - 2003, and at Carroll College 2004 - present, so I have seen a lot of teams succeed and others that didn't do as well as they could. The advice that follows comes from my own personal experience both in the contest and as a coach. This is how I would attack the MCM today.

Kelly S. Cline

# 1 Anatomy of a Paper

## 1.1 Summary

This is without a doubt THE most important part of the paper. The difference between an honorable mention and a successful participant is that the judges probably never read much more than the summary of an SP paper. The summary should be written LAST. Let me say that again: You should not write the summary until the rest of the paper is done. In an ideal timetable, all of Monday should be set aside for writing the summary.

The summary must briefly hit all the main points and ideas of your paper. If you did anything creative, it must be here. Further you should put numerical results in the summary: “Our final algorithm performed 67.5% better than a simple greedy algorithm, and 123.3% better than a random choice”.

Ideal summary length is hard. You must include all your main ideas in the summary, but brevity is VERY important. I would try to make the summary around half a page, definitely not more than 2/3.

The summary (and ideally the whole paper) should be written collaboratively, as a team. Here’s my best advice on how to do this: Break up in to different rooms and have each person sit down individually and write the best summary that they can. Set aside plenty of time for this, maybe an hour. Then, come back together and read the summaries out loud to each other (out loud is very important). After discussing them, set them aside and as a team write a new summary together up on a blackboard.

## 1.2 Introduction

In the introduction, you want to restate the problem as you interpret it. Within many MCM problems, almost every team will find a different ‘problem’ to work on. After the contest if you read the papers from other teams, you will be amazed by how differently they approach the problem. Often they won’t even be working on the same thing!

So what you need to do in the introduction is to clearly explain how you interpreted the problem, and what you decided to work on. Writing the introduction and having the whole team read it can be a very good way to make sure that everyone agrees on what the problem is, and what needs to be done. The introduction is also a place to give a little more background on the problem, and show what you learned while researching it. Remember, the people reading your paper are math professors – they’ll get offended if you don’t show that you understand the traditional textbook approach to your problem. Whether you chose to use a textbook method or something more creative, always mention the traditional methods in the introduction, so they know you did your homework.

The introduction can usually be written first as a Friday project. It can help make sure that all the team members are in synch about what they are really working on.

## 1.3 The Model

The purpose of a mathematical model is to predict how some real world system will behave, and to help you to understand it. The first big section of your paper should be to describe your mathematical models. Most problems can be broken down into three parts: the models, the solutions, and comparison methods. You’re given some type of goal, and you’re asked to develop a method of achieving this goal: Finding a submarine, catching a prey dinosaur, get

customers through an amusement park as quickly as possible, evacuating people away from a hurricane. The purpose of a mathematical model is prediction. Its purpose is to predict what will happen if you do different things. For the submarine problem, you would describe the methods of modeling waves traveling through water. For the velociraptor problem, this models dinosaurs stalking and chasing each other. For the MRI problem, this part of the paper would describe how you created simulated data representing biological tissue. For the Hurricane Evacuation program, your model would predict how long the evacuation would take.

Good papers generally contain a series of models, starting very simple and working toward more complex and realistic models. You should always try to figure out a way to create a first model so simple that you can solve it on paper. Generally, more complex modeling will occur on the computer, so the challenge is to translate the computer work into words, and justify each step. In order to create these models for continuous problems, I would recommend having a clear understanding of how to solve differential equations, however many continuous problems have not involved this. For the animal population problems, you would want to be able to write differential equations describing the relationship between the predator and prey populations, then numerically integrate them. Know what a partial differential equation is – know the wave equation and the diffusion equation and what they mean. With this knowledge in hand, the concrete slab problem would have looked almost like an old friend!

Remember, this is the mathematical MODELING competition, so do not gloss over this section. It may be simple: For the grade inflation problem, this section might simply involve simulating the actual grades for a class, perhaps using a random distribution, then using some method to skew them upwards due to inflation. In general, for the discrete problem, you want to be familiar with how to generate random number sets with different properties – this can be very useful in constructing the sets to test your solution methods with. The computer person should be setting up these models Friday morning, and so this section should be roughed out on Friday or Saturday.

#### **1.4 The Solutions (PLURAL!)**

The second BIG section of the problem. Here, we describe our methods of trying to achieve the goal: We will attempt to protect our stunt person by cushioning their fall with four cardboard boxes. We will attempt to undo the effects of grade inflation by ranking the difficulty of different classes, and giving difficult classes a greater weight in GPA calculations. This is the section that actually describes how we solve the problem. In the submarine problem, this is our algorithm that takes the simulated data from waves traveling through water, and uses this data to guess at the position of the submarine. In the velociraptor problem, this is our algorithm which mathematically states how the raptor tries to catch the thescelosaur, and how the thescelosaur tries to get away.

(Occasionally a problem will explicitly give you the solutions they want you to test. In the Escaping a Hurricane problem, the solutions were to reverse lanes on the freeways and possibly the surrounding roads. Other times, no solution is really needed at all: The Deep Impact problem simply asked teams to predict the results of a large asteroid impacting Antarctica.)

You **MUST** have more than one solution. Let me say that again: **MORE THAN ONE SOLUTION**. In order to show that you have a brilliant method of finding submarines or cross-sectioning gridded MRI data, you need a baseline, something to compare your solution with. You want to start with the simplest, most obvious algorithm possible, then gradually

build on it, refining it until get to your best solution.

Often for the discrete problem, the simplest solution may be merely to make random choices. For the meeting scheduling problem, you might want to have one algorithm which just randomly makes up the schedules. Then when you compare your better solutions with it, they look good!

You want to show that you've explored the problem thoroughly, and that you've tried many different approaches. Even if you started with your best algorithm, then tried a bunch of blind alleys, in the paper you want to present things as if you started with the dumbest most basic solution, then gradually refined it and finally arrived at your best solution.

What if you tried a more sophisticated solution method, which didn't work well? Put it in the paper! Show all the angles you tried, even if your best solution is not the most complex and interesting one. In real life, that happens very often!

## 1.5 Solution Comparison Methods

Usually the problem will state very clearly what the goal is, so it makes your algorithm testing methods fairly easy. For the submarine problem, your model is to create a simulation of sound waves propagating through water, bouncing off a submarine, then being received by an array of microphones. This data is passed to your various solution methods which all take a guess at where the submarine actually is. All you have to do is find out how far each algorithm was from the mark and you have an easy method of comparison.

However, you usually have to make some decisions in how you compare the results of your solutions. In the MRI cross-sectioning problem, you can compare your algorithms' estimate of tissue density with the actual density created by your models for each of the thousand or so points. But do you just average the variance? Maybe you should look at RMS error? Are you concerned with making sure that no point is drastically wrong or that the overall error is small?

With a lot of problems there will be many ways to compare your different solutions, and there's good reason to use more than one method to evaluate them. Evaluation methods should be one area of brainstorming that you keep working on all weekend.

## 1.6 Results

Here, you need to actually present the results of the testing. This section should be very focused, because you've described everything else. If possible, you want a lot of data to back up your conclusions. Try a variety of models, and use them all to predict the results of a variety of solution methods. In general, you're going to end up with a lot of parameters to play with – in models, your solutions, and comparison methods. Try to explore as much of this parameter space as possible. You want to show that you've taken a mature approach to the problem, and probed all aspects of it as best you could.

The specifics of data presentation are difficult. If you can make graphs, by all means do so. In the MRI problem, we made residual plots for each of our algorithms and each of our data sets. But ultimately if you've thoroughly explored the parameters of your models, solutions, and comparison methods, you're going to have a huge quantity of numbers to present.

Give them the numbers from all your results in tabular format, but don't expect them to read the tables. You want to create a narrative in the text, going through the tables, pointing out the general trends, the exceptions to the rules, and the overall results.

**IMPORTANT:** Many teams create a model, a solution, run a test of their solution, present the results, and stop. You must run **MULTIPLE** tests! You must find out if your solution is stable! Will it hold up under slightly different circumstances? At this point your computer is behaving itself and generating useful information – take just another 20 minutes and try a few more cases, varying the parameters just a bit. If you’re doing the Escaping a Hurricane problem, try things with a few more cars, or a few less – see how flexible your results are. If you’re doing the MRI Problem, test your routines on several different simulated organs. If you’re doing the Air Traffic Control problem, throw the planes at your simulated airport a bit faster, a bit slower, or take a runway out of commission and see how your solution copes. Prove to the judges that your solution is flexible and stable, or honestly admit that your solution doesn’t work as well against certain challenges. Your paper will look a whole lot better!

## 1.7 Conclusions – S&W – Future Work

First, present the bottom line, even if you just presented it in the Results section. ‘Overall, solution method A performed 34% better than B, and 67 % better than C.’

You need general numbers which summarize everything, so figure out a way to somehow average all the data and distill this into a few numbers which numerically rank the algorithms. The results section is where you say ‘Strategy A worked better in these circumstances, but B had the advantage when we did this.’ In the conclusions section, you have to boil it down and say ‘A is best.’ This is also what you will need to put in the summary, so be sure you get specific overall results.

Strengths and weaknesses is a great way to go over the key things – explain the good and bad points of your algorithm. I like to use a bulletized list. Nothing new should go into S&W – you are summarizing. The main points of your results section should be here, along with the shortcomings, and any limiting assumptions as well.

In order to show that you are mature in your approach to the problem, a future work section may also be useful. What would you do if you could work on this problem for the next few months? Were there any algorithms that you thought would be great, but just could not implement on the computer? The MCM is very limiting, so this is a place to show that you can see the big picture.

## 1.8 References

Don’t forget to reference any resources that you use! You’re going to spend a lot of time digging through books and web pages looking for things that work: Every time you find something **MAKE A NOTE!** You’ll come up with some original ideas of your own of course, but most of what you’ll do is take existing ideas and adapt them to your particular problem. In your paper, you **MUST** include references in the text like (Simmons 2002) and then include a full list of references at the end. When you use someone’s idea without including a reference, that’s called **PLAGIARISM** – which is a really bad thing! Even if you’re not stealing words, but just ideas, that’s still plagiarism! The judges will do some rather nasty things to your paper, if they think you’ve committed plagiarism, so don’t do it!

If anything, pad out your list of references to show just how much research you did! If you found some unusual source that other teams probably didn’t locate, include it as a reference – that makes you look good! Careful referencing makes your paper look a lot more professional, which is your main goal in writing the paper!

## 2 A Skimmable Paper

The first thing to keep in mind is that the judges are looking at a lot of papers in a very small amount of time. In their first pass, (in which half of the papers are thrown into the successful participant pile!), the judges can only give FIVE MINUTES to each paper. This means that they'll read your summary, skim through the rest and then your paper gets tossed in one of two piles. That's it! Half of the papers never make it past this stage. Even worse, in the second round of judging, your paper will STILL get only about FIVE MINUTES. Only by getting through the first two rounds will your paper be read in any detail. This means that your #1 goal is to beat the five minute read.

We've talked about the summary (THE most important part of the paper), now what about the skim? Things that may get noticed are headings, bulletized lists, tables, diagrams, and figures. What you don't want are large unbroken blocks of text, they make the paper look boring, and may never be read completely. You want the words to be clear and easy to read, and the text to be broken regularly by headings, lists, figures, diagrams, anything you can think of to keep it interesting.

### 2.1 Headings

Headings are very important. If you take out all of the text of your paper, the headings should read like an outline. The judge should be able to read just the headings and see the flow of your paper (which should match the ideas presented in your summary). You want at least a couple layers of headings (heading, subheading, subsubheading) that easily breaks your paper into small pieces, each with a clear purpose and goal. Try not to go more than a paragraph or two without a heading. It not only makes your paper skimmable, it helps the paper stay focused, keeping the paper from wandering. As an example, here is the outline of my team's outstanding MRI paper.

- Summary
- Introduction
  - Assumptions
- The Models
  - Coordinate Systems and Definitions
- Interpolation Algorithms
  - Method 1 – Proximity
  - Method 2 – Density Mean
  - Method 3 – Trilinear Interpolation
  - Method 4 – Polynomial Interpolation
  - Method 5 – Hybrid Algorithms
- Testing and Results
  - Overall Results

- Strengths and Weaknesses
- Future Work
- References and Supporting Data

## 2.2 Bullet Items

By this I mean:

'To set up this model, we included 4 key ideas'

- First idea
- Second idea
- Third idea
- Fourth idea

These types of lists, whether numbered or not, have three really important purposes.

1. They break up blocks of texts, making reading less tedious.
2. They emphasize important ideas.
3. They are easily noticed when skimming.

## 2.3 Tables of Data

If you get a good computer model working, don't waste it! Run it a few hundred times with all kinds of values of your input parameters (submarine depth, concrete slab size, whatever you can think of). Then organize the data in graphs (if you can) or tables. They can be very impressive even if the judge doesn't read them in detail. They show that you have lots of data to support your conclusions and you've explored the parameters of the problem thoroughly.

## 2.4 Diagrams and Figures

A picture really is worth a thousand words. Diagrams can be very useful in the model section to show how you set up the problem, and figures are always the best way to display data. When my team did the velociraptor problem, I think one of the best things we had were clear diagrams showing our mathematical models for the dinosaurs and how they moved. Describing their allowed trajectories in the paper was very difficult, and I can't even begin to recall how we phrased it – the shapes are geometrically rather complex. However I could sketch out our diagram in a couple of seconds and you would understand the idea completely.

## 2.5 Backwards Design

The most straightforward way to create your paper is to start with the problem, work out your series of solutions, then write your paper explaining your solutions. However, it's also very useful to turn things around: Even without thinking of your solutions in detail, try to design what you want your paper to look like, and spend your time working to fill it in. Essentially this is what I've been doing, sketching out a paper in the most general sense. Once you've



looked at the problem, you'll be able to do much better, and my general sketch here may not fully apply. Sometime early in the contest, write an outline of what you think the 'ideal' paper should look like. Find a blackboard and outline the major sections and subsections. Make quick sketches of all the figures and diagrams you can think of that might be useful. Brainstorm things to do bulletized lists of and other things that can go into the paper.

Make sure you all look at this outline and discuss it – it will help you all stay focused on the same goal. When you're down in the trenches trying to code out some algorithm or analyze some data it's easy to lose sight of the big picture and spend several hours trying to do something that's not completely essential. By keeping the outline of your ideal paper in your mind, you can let the outline direct where you spend your time. There WILL be some things that you want to do that would look great in the paper, but may just be too hard to do for one reason or another. By working out your paper, even before you've worked out the solutions, you can stay focused and do only the work that needs to be done to make the paper look like it should.

## 3 The Team

I'm including this section not because there is one right way to break up the responsibilities during that contest – that's silly – but to try to get you to think about what role you will fill, what responsibilities you can assume: to show you one way to form a team that happened to work. This is how we broke things up when I did the MRI problem and it was very effective. When you tell one team member that their #1 responsibility during the contest is to write the paper it assures that writing begins right from the beginning and that lots of time is spent on rewriting and revision – not just Monday afternoon! Assigning a team member to take charge of computer work can also be very useful. It is possible to do some problems purely on paper, but the MRI problem was one which demanded a lot of computer work, so if possible you should go in with at least one person ready to handle that area. Therefore I think this way of assigning roles is not completely arbitrary.

Right now, think of what role you can fill, and how you can best prepare.

### 3.1 Computer Person

Most successful teams do a lot of computer work, and this was the role I filled in my second two attacks on the MCM. This could mean programming in a language like C++ or Java, or it could mean setting things up in Mathematica or Excel. If you've got someone on your team who knows how to program in C, that's good. If not, use the skills that you have.

Most complex models require a computer to make their predictions. The computer person must first implement the mathematical model so it is capable of making predictions, then set up the solution methods to tell the mathematical model what we are going to do, then deal with the resulting data and compare how different solution methods work out. You'll never know what works or what doesn't until you actually get through the third step in the process. This will often involve a HUGE amount of time in front of the screen. Before the contest, make sure that you know what computer tools you'll be using, and that you're comfortable with them.

Books on numerical methods can be very useful, depending on the type of problem that you get. One very good one is called Numerical Recipes in C and the entire book can be downloaded off the web in pdf: <http://www.library.cornell.edu/nr/cbookcpdf.html>

Visualization is also very important. Make sure that you're ready to make lots of plots and graphs, in Excel, Mathematica, or whatever you're using.

### 3.2 Writer

The paper is what your team is judged on, period. If it's not in the paper, it doesn't matter.

The paper must be written very, very clearly. It must say things very simply and precisely. You do not need to make things sound complicated – anyone can do that. Your program is already more than complicated enough. As writer, your goal should be to make everything as simple as possible. Don't write in that phony 'The data were collected.' style. Use personal pronouns: 'We collected the data.' This makes the writing much more active, and much less dry. Also, the key to interesting writing is often a good verb. If you've got a sentence that's dry and boring, try to find the action in it – what HAPPENS! – and try to find a good verb to rebuild the sentence around.

READ THE PAPER OUT LOUD! Let me say that again: read the paper OUT LOUD. You never know how things sound until you actually hear them – it makes problems so much

more easy to spot. If things are even the slightest bit confusing, then they must be changed. Remember, you're in the middle of the competition – if you don't quite understand it, the judges won't have a clue.

In the end, I think that paper writing should be a completely team project, but someone has to take the lead. The writer's goal should be to create a thorough draft of the paper by noon on Sunday, covering everything.

You must get everyone to critique every part of the paper, and one of the hardest parts is learning not to take that criticism personally. Writing without an ego is difficult, but that's what the MCM requires. As writer, you need to get chunks of the paper written, then circulate them around, get feedback, and go back and write again. Be comfortable with writing, and rewriting, and rewriting over and over again.

As preparation, you need to do a whole lot of reading. Read every MCM paper you can get your hands on, especially the outstanding ones. The outstanding papers in the UMAP Journal will have a judges commentary, which is EXTREMELY important. This is the only place where you actually get to look into the mind of the judges and see what they are thinking. After you read an outstanding paper, try to predict what the judges will say – do your best to get inside the judges' heads! Get a notebook and jot your ideas down, looking for things that you like and things that you don't. You need to know what makes a good paper!

It's also probably the writer's responsibility to get familiar with whatever word processing package the final paper will be written on. Be sure you know how to do equations, headings/subheadings, bullet lists, etc. I'm partial to LaTeX, but Word has a good equation editor. Make sure you know the basics.

### 3.3 The Third

Writing is the cornerstone of every attack on the MCM, and computer work can be almost as important, there are many other important tasks. In order for a team to win, all three people must be working at their peak, so the third must look for things that need to be done.

The first big hurdle is research. Find out as much as you can about the problem, and possible ways to solve it. To be useful in the MCM, research has to be very specific. Dig through math texts looking for specific things that can be implemented on the computer. Get to know Numerical Recipes and Numerical Methods that Work really well, and look for things you can use.

*Research Hint:* The MCM has been running since 1985, which means that there have been more than 60 problems posed since the beginning. The Outstanding papers for these problems have been published in the UMAP Journal. It may be useful to quickly skim through the past problems and see if there has ever been a problem similar to yours. If so, look up the outstanding papers for that year and see if they contain any ideas that could be adapted to your problem. Maybe there's nothing relevant, but who knows? It's worth the effort to spend an hour looking: You just might discover a key idea!

After research, make sure to get involved in writing as well. Be a sounding board for what the writer has done, and make sure that everything is very, very clear. Everything should flow and sound completely natural. Don't just be a reader of course. This should be a collaboration all the way – everyone should write.

Computer stuff – Are you good with computers? If not – learn! One of the best things any team can have are two people with good computer skills. When I did the MRI problem, I had one team member who started out not knowing C at all, but by the end that had changed.

Get in there and learn as much as you can.

If you know a bit less, then you need to be around for the data collection phase. Sometime on Saturday (or Sunday at the latest), the computer model must be frozen, and you need to start generating results. The model has been set up, and most of the solution methods are done, so it just takes a fair bit of time for all them to be tested and compared with all the methods.

If your team is writing computer programs, and the programmer is the only one who actually runs the programs, then the team is in SERIOUS SERIOUS trouble. In the MRI contest, I worked at the programs until I thought they were running perfectly, then I gave them to the third, explained what he had to do, then told him to start generating data to fill up the tables. Naturally, the code that worked perfectly for me broke within seconds for him, so I found the bugs and gave things back to him. After about three cycles of this the programs stopped bugging out, and we started generating usable data that ended up in the final paper.

the code out of the programmer's hands and generate the actual data which will go into the paper. In my MRI team, all three of us ran the programs, although the writer had almost no programming experience to start with.

### 3.4 Teamwork

One dangerous pitfall that many teams fall into is that when a disagreement comes up, they vote, two people on one side, one on the other, the majority rules and the thoughts of the third person are ignored. Several times I have seen a team member frozen out in this way, who then feels disenfranchised and left out. Instead, decide right from the beginning that voting is off the table: No voting allowed. Instead, all decisions need to be made by consensus or not at all. If there's a 2-1 split then you need to sit down and talk things through, and the two need to listen very carefully to the thoughts of the third, to understand where he or she is coming from, and to figure out what the other two are not seeing.

Another pitfall that can come up when you have a disagreement is for the team to break up and go in different directions: "We think that the best strategy is A. You think the best strategy is B. We'll follow our strategy, you follow yours, and combine our work tomorrow." This is a recipe for disaster: You need to have all the members of your team working together on one coordinated strategy. Sometimes I think this happens because a team is unable to agree on a strategy, so the two simply tell the third to go and work on some other idea, just to get that person out of the way. If your team disagrees, you need to sit down and talk until you reach agreement, instead of splintering into different directions. Even worse, sometimes the two try one approach and write one section of the paper while the third tries another strategy and writes another section of the paper, with the idea that they simply be cut and pasted together at the end. This almost always results in an incoherent paper that makes no sense. In the worst cases of this, I have even seen teams where at the last minute the two cut their teammate's work out of the paper entirely in an effort to salvage things. This is a completely unacceptable and just plain mean thing to do: This person has spent several days working hard on something and writing it up, only to be out-voted and find that all that work was completely wasted, and was not a part of the final result. So here's the rule: No one is ever allowed to cut another person's work out of the final paper without their permission. You must all work together on every aspect of the paper, and everyone must agree at every step. First and foremost, don't splinter to begin with: Sit down and talk things out until all

three members of your team all agree on one coordinated strategy.

It's always important to spend time listening very carefully to your teammates. Usually the most talkative person ends up being the team leader, and more quiet students are more easily ignored. However being talkative has little to do with intelligence, and some very smart people are quiet and a little bit shy. You need to listen, to make sure that everyone is participating, and if someone on your team isn't speaking up, it helps to specifically ask for your teammate's opinion.

As a general principle, I recommend that you avoid absolute language when talking with your team. Rather than using phrases like "This is it!" "This is right!" "This will never work!" that tend to close off issues and shut down discussion, it's a lot more useful if you can phrase things cautiously and tentatively: "Maybe we should look at this." "I don't see why this makes sense." "Perhaps we could try that." Cautious language opens up discussion, and encourages others to voice their own thoughts.

During the contest you should never find yourself out of a job, with nothing to do, picking away at some peripheral detail that will never make it into the final paper. If you ever find yourself spending time on something that is not really important, not really contributing, then be sure to recognize this as a huge warning sign. A successful team must have every person spending every minute on something really vital. This goes especially for more junior members of a team. If you're a freshman or a sophomore, and you're on a team with an upperclassman or two, it may be harder to find ways to contribute, to stay in the thick of things, and to add important ideas when everyone else seems to be an expert on all this. You must keep yourself involved because there is far too much work for them to do this without you, or with you idling along at half throttle. If you find yourself in this situation go to your teammates and tell them point blank: "What should I be doing? How can I contribute?" If you're a more senior member of a team, then don't take everything on your shoulders. Work *with* your teammates and don't be afraid to delegate. My biggest problem the second time I did the contest was that I tried to do everything important myself, carry the entire load, and I utterly refused to ask my teammates to do anything really important. We got an honorable mention, but I know we could have done a lot more if we'd been working as a team, instead of me selfishly hoarding everything vital all for myself.

So if you're a freshman or a sophomore, what should you do? How can you contribute? What role should you play? First you must make sure that you understand the math that your team is using in every detail. Suppose your teammates come up with some really cool algorithm to solve the problem: Step one of explaining their ideas in the paper should be to explain things to you. Make them explain things until they become absolutely crystal clear. Remember you have probably the freshest perspective on things, so whether or not you are the official team writer, try to act as a gatekeeper for material to get into the paper. Look for things that don't make sense, that are vague, that are fuzzy and unclear, and then make a big stink about it until the smoke is blown away. A freshman is in a great position to be the official team skeptic. Ask lots of questions, every question you can think of. Why did you do this step? What does this mean? Where does this equation come from? Your final paper should be written in much the style of a textbook, it should be written in order to teach someone about the methods that you used. Of course the best way to evaluate a textbook is to give it to a class of freshmen and see if they can understand it at all, or if the whole thing is just a confusing jumble of jargon. Don't let your teammates snowplow you: If they can't explain what they're doing to you, they don't have a prayer of explaining it in the paper.

## 4 A Timetable

### 4.1 Before the contest

Together you should read lots of problems. Be comfortable with brainstorming together. After reading a problem, see if you can break it down into the three parts: What needs to be modeled to generate data? What will your solution algorithm need to do? How can you compare algorithms? Make sure that you are comfortable together, and that when you're brainstorming everyone participates, and you don't have one person dominating.

Practice writing together too – that's even harder than brainstorming. See if you can write as a team with everyone contributing equally.

### 4.2 Thursday Evening

Spend five or ten minutes reading both problems carefully. If everyone agrees which problem to do right off the bat – go with it! The more fun and interesting the problem is to you, the easier it will be to work like hell on it. One of the hardest choices is if you have one problem which looks totally cool and one which is less interesting but you think you could do a better job on it. Given my own experience and that of people I know, I say go with what is most interesting – you'll be more motivated, and you may surprise yourself. Remember, this contest is not about what you know at the beginning, but what you have taught yourself by the end.

Spend about twenty minutes or so brainstorming. This means

1. Set a loose time limit.
2. Appoint one person to write down all ideas on a blackboard. If each person writes down their own ideas then they become personal property. Instead every idea should be public.
3. Get every idea you can think of on the board.
4. Try to build on the thoughts of your team mates.
5. No criticism or negative comments allowed! That's for later.

After brainstorming, go back over things in a more organized manner. Break each problem into the three main parts: What are you modeling? What ideas for algorithms do you have? How will you compare the algorithms? Every problem has these three components, and you **MUST** be able to see exactly what they are. Without breaking down the problem, you can never get a foothold on it. Also, breaking down the problem into these standard pieces keeps you from neglecting a potentially important area. Spend some time discussing all three sections, and make sure you all agree on precisely what the three parts involve.

Look for other, less generic ways to break down the problem. You want to chunk out the problem into manageable pieces that can be attacked one at a time. Can you break the problem up using a timetable – a certain sequence of events? What about different classes of strategies?

Identify whether the problem is well focused or very diffuse. If the problem is very big and broad then you have a lot of choice in what problem to do, and how to formulate this in mathematical terms. Later when you read papers from other teams, you'll be amazed that the teams weren't really doing the same problem! Remember this throughout the contest: You are not only solving a problem, but creating the problem as well. When I did the submarine

problem (the grand-daddy of all diffuse problems!) we formulated things in one way, then found that we couldn't solve it, and spent the rest of the contest banging around that blind alley. If it's very diffuse, there may be hundreds of possible problems that you can chose to solve – chose the one that you can solve in the best and most interesting way.

That being said, some MCM problems are well focused and you don't have a lot of options – there really weren't too many ways to approach the MRI problem. Right from the beginning, try to recognize how focused the problem is and what choices you have.

At this point you should make a tentative decision about which problem to do. If you focus on one problem the whole time then you will avoid wasting resources. The next step after deciding on the problem is research. Hit the library and find as much information as you can about all aspects of the problem. Get the resources and find out what is there and what needs to be read.

Each time I participated, I held the record for most consecutive hours without sleep. (I don't remember exactly how many I managed – things got kind of fuzzy near the end.) But you know your own limits. This contest is about pushing yourself to the limits, but don't go beyond them. This isn't about making yourself sick! You want to focus every lucid moment of the day on the problem, but you also need to maximize the number of lucid moments you have!

After some thinking and perhaps a quick look at a few useful textbooks, the computer person should start setting things up – either Thursday night or Friday morning. You have three huge pieces to assemble and you need to get started as soon as possible. Usually the model itself can be done first – get that running as soon as possible. Get a good head of momentum going and go for as long as you can stand.

### 4.3 Friday

On Friday, the writer and third should begin with lots of research. Most MCM problems can be seen as specific examples of a general type of problem – MRI was an interpolation problem, submarines and the concrete slab problem had partial differential equations. Find out what type of problem you're doing, and learn as much about it as possible. Go to school. Read textbooks. You have to teach yourself everything you can on the subject. When we did the MRI problem, all three of us put each other through a short course in interpolation methods.

Papers work out best if you can use some established 'textbook-type' math to solve your problem. When I did the Velociraptor problem, I think our biggest weakness was that we failed in the research side of thing, and ended up having to create all the strategies ourselves. Later, in reading other problems I discovered that other teams had found ways to incorporate geometry, game theory, and differential equations into their solutions – and the power of the established mathematics made these solutions incredibly effective. Sometimes it's obvious what kind of math you can use (interpolation methods for the MRI problem). Other times you will have to take some vaguely related area of mathematics, and shoehorn it into the problem, trying to make it fit as best you can. This should be a big research goal – find something established that you can apply.

The writer should start on a draft of the introduction as soon as possible. If you get that done, keep going, rough everything out as thoroughly as you can. It is your job to keep in touch with the computer person – make sure everything he/she puts into the machine actually gets into the paper. Get everyone to read everything you write and get lots of feedback. By Saturday, research should be over, and you should be rewriting and rewriting. This is all

subject to the whims of the problem of course – in the MRI problem, on Saturday both the writer and the third were running the code generating final data because the paper was looking pretty good.

Keep in touch with each other! Don't get fragmented! Check in with each other every few hours and explain to each other what you are doing. If you feel like you aren't doing something important or that you are wasting time SPEAK UP! You are responsible for making sure that every minute of your time is spent on something vital! If it's not, then find something to do! This is VERY VERY important!

When I did the Velociraptor problem I had one team member sitting largely idle throughout much of Saturday. The person was working, but not on anything which would go into the paper – and I think this really hurt us. If you see this happening, take action! Don't be afraid to speak up and find something for this person to do – You must use all your resources if you want to win!

#### 4.4 Saturday

Sometime on Saturday, the computer work should be done and you should start generating final results that can go into the paper itself. If your team has written up a program, my experience is that complete debugging of the code will never happen until someone other than the programmer actually runs it. Hopefully by this time you will have played around with the code sufficiently to spot-check your algorithms, but at some point you have to start generating real data which goes into the paper.

Once you get your final results (reversing the freeways allows South Carolina to complete a hurricane evacuation 29% faster), this is the point where you need to run a few more cases to demonstrate the stability of your work. Vary the parameters, the data sets, the number of airplanes, cars, or dinosaurs. Are your results very sensitive to these things, or do they not make much of a difference? It is very important to find out, and to show the judges that you've really put your model and solution through a thorough test.

Also on Saturday, my feeling is that the writer should almost finished writing. The bulk of the paper should be fleshed out by this point – all the necessary ingredients should be there. In order to see writing objectively, you have to look away from it for a while, so I advise the writer to go help with data collection for a while.

One particular worry sometimes pops up at about this time: You've got your teeth into the problem a bit, you've got something running, and you've got some results, but you're worried that you haven't done anything that every other team hasn't done too. You may be concerned that you've only done the simplest, most obvious things to do, so there is nothing to set your team apart, nothing to really give you that extra edge in this grand competition. If you and your team find yourself worrying about this, I have three suggestions:

1. First and most importantly, whatever type of math you're doing, do it well, do it precisely, do it carefully, do it right, make sure that you understand this corner of mathematics at the deepest level, and then explain it so well and so clearly that the concepts just melt off the paper. The best papers are often ones that do something fairly simple, fairly basic, but they do things and explain things so carefully that it is obvious that they really understand the methods that they're using. That is the mark of a great paper: Use the right mathematical tool for the problem, and make it clear that you understand this tool inside and out, its strengths, its weaknesses, its limitations, why it is useful, and where it will fail.



2. You will often find that your problem is a specific type of a general problem (e.g. a ‘knapsack’ problem, an interpolation problem, a ‘traveling salesman’ problem, a differential equation problem), and that a lot of smart people have worked on this type of problem so that there are well established methods for dealing with it. So learn these methods thoroughly, explain them well, and then *customize* them to your particular problem. Read the problem statement very carefully looking for the specific details and idiosyncracies that make your problem unique, that distinguish it from the general case. I think this is what really put my MRI team over the top: We recognized right off that we had an interpolation problem, so we hit numerical recipes and a couple of textbooks to bone up on what interpolation was and how it worked. Then we noticed that in the problem statement they said that they didn’t want to blur out the boundaries between different types of tissue. So to customize our algorithm, we put together a method for finding these boundaries, and turned off our interpolation routines whenever they hit a boundary. I think that was what made our paper stand out from the crowd. We found the right tool (interpolation), we did our homework so that we could explain it in crystal clear detail, and then we adapted this method around the specific details of our problem.
3. Of course it never hurts to strive for some real creativity. Saturday is probably a really good time to go back over your initial brainstorming and hunt for other methods and angles on the problem that you have not pursued. Get your team back together and try to do some more brainstorming from scratch. Try to see if your problem might fall into more than one general category: Could this be approached as either an interpolation problem or as a differential equation problem? That could make for a truly awesome paper if you could show how two very different mathematical tools could be adapted to the same particular problem, allowing you to compare and contrast their effectiveness and their limitations.

#### 4.5 Sunday Morning

Sunday morning, tie down the loose ends and GET THE FINAL DATA! Work on visualization, graphing, making pretty pictures etc. You MUST have virtually ALL the data that will go into the tables in your paper by Sunday at noon!

The hardest thing about this contest is knowing when to stop. You’re involved in this because you enjoy a challenge. You want to do more than just turn in your homework and get a passing grade – you want to really understand what’s going on. You’re tired of the wading pool: You want to swim in the ocean. But that means it will be very difficult for you to say “This is far enough,” to end your work, stop programming, stop making figures, and focus your whole team on serious writing.

At the end of the contest, all you will submit is your paper, and there’s one secret to writing a good paper: Spend a lot of time on it. If you spend a lot of time writing your paper, it will be a good paper. If you spend only a little time, it will not.

So here’s the hardest thing, perhaps the biggest challenge you will face during the contest: Can you set a deadline for Sunday at noon, when all work must stop, and everyone puts 100% into writing the paper? Can you stick to this deadline? Can you stop when you are just ten minutes away from some great new data? I have known only a few teams with the strength to hold this deadline, and they have all produced excellent papers. After you’ve written the paper together during Sunday afternoon/evening, you can go back and get those

last few things, and add them to the paper. Think about this very carefully: Can you set this deadline? Can you stick to it?

## 4.6 Sunday Afternoon

Okay, in my opinion, here's what put my MRI team over the top and won us the outstanding. On Sunday afternoon we all stopped what we were doing, we read over the draft of the paper, discussed it a bit, and then we threw it away.

Together, we spent the next few hours writing our entire paper on the blackboard. The draft written by the writer at that point was very good, and it made sure that all the important ideas were in the air and that nothing was forgotten. However, no paper written by one single member of your team will ever be as good as a genuine collaboration.

We went through the paper from beginning to end, writing it all as a team, up on the blackboard. We argued over sentence structure and word choice, sweating over each phrase for several minutes, reading it out loud, trying to find a better way to say things. In some places, the existing draft was excellent, and we copied this up on the board – but putting it on the board took it away from any one person and gave the paper to all of us. We filled up every chalkboard in sight, and that Sunday afternoon/evening was what put us over the top.

When you're writing the paper like this, make sure you're all relaxed and comfortable, and don't try to get things done quickly so you can get back to your code. THIS is the single most important task you will do during the entire contest. Relax and don't be afraid to express your thoughts. Each of you should be talking, suggesting ideas, bringing up things you need to include, complaining that a sentence 'doesn't sound right.' If only one or two people are doing the talking, then something is seriously wrong. If any sentence sounds even the slightest bit confusing to you SPEAK UP! Each team member will have a very different perspective on things, and only together can you write a really good paper. Every sentence in the paper should be spoken OUT LOUD at least three times before you go on to the next! Also, regularly trade off who holds the chalk – it helps keep everyone involved.

Okay, by the time you all crash on Sunday, you should have the entire paper worked out. You may or may not have the summary done, but the rest should be complete. Yes you might want to gather a little bit more data for your tables, or find a better way to make a picture of some data, but by Monday morning, the contest is just about over – computer work should cease!

## 4.7 Monday

Monday morning should be reserved for writing and rewriting the summary. Again, my best advice is for each of you to go off to different rooms, and write out the best summary that you can by yourself. After perhaps an hour of working on these individual summaries, join up again, and read these summaries out loud to each other. Talk about them, and use these as the starting point to writing your group summary.

If you agonized over every word of the rest of the paper, here you should be sweating blood! Find a blackboard and put your summary on it. Don't try to do this on a computer screen!! You MUST make the summary community property. Go over it and over it and over it. Make the summary a work of art, and try every possible way of structuring it that you can think of. Spend time discussing what should and shouldn't go into the summary. Remember, the summary is THE most important thing you will create.

After your team has a summary that you all like, this is a great time to go back and read the paper again. Read it carefully, out loud if possible, looking for problems, typos, awkward grammar, or confusing sections. Now is your last chance to rewrite and polish things up. The most successful teams that I have known have spent a few hours on Monday (after the summary is finished) just reading the paper, combing it for places to smooth out and improve.

All work should be done by 3:00 pm on Monday, three hours before the contest ends. At this point, stop and print out the final version of the paper. Just for the record the printer will jam, the computer will crash, and everything that can possibly go wrong will go wrong on Monday afternoon. The danger of losing the competition just because you could not get the paper printed out in time is very real. At 3:00 pm on Monday, drop everything, and work as a team to get a final version of the paper printed out.

After you have a final version, if there are one or two things you want to fix – great, fine, whatever. See if you can make another final version (or fix a couple of pages), but do not even think about it until you have a paper which you could seal in an envelope!

*Potential Pitfall: Monday Burnout* Several times I have seen teams finish the contest early, and usually the results aren't great. It's Monday at 10am or noon. You and your team are sleep deprived and burned out beyond all recognition. Your paper is at a stopping point. It's not perfect, and you know it, but you don't have the time to do anything major. Theoretically you have another 5, 6, or 7 hours, but you are so fried that you can't think of anything useful to do with this time. You've done your best. The paper is what it is, and you want to just turn in the paper, be done with it, go home and sleep. If you find yourself in this position, let me recommend a different course of action. A few hours at the end of the contest really can be important, they really can make a difference. Instead, I recommend that you print out a potentially-final version of the paper, then send everybody home for, say, two hours, to take a nap. It's noon, everyone agrees to go home and try to sleep/rest/whatever, and then come back at 2pm. There's all kinds of research saying that even very short naps have huge effects improving your thinking ability. After a couple hours, come back together and read the paper out loud to each other one more time. I promise that you will be amazed at how clearly you will see things, and how many problems that you can now identify and fix. If you've got the time, try to make every hour count, and see if you can go the distance!

## 5 The Time Trap

So far we've focused on the right things to do – now let's look at the other side of the coin. I've been on one successful participant team that really crashed and burned and one that barely squeaked up into an honorable mention. The fact is that half of all papers are ranked successful participant, and most of them deservedly so. There are of course many ways that a team can fail, but there is one pattern that I've seen repeated, both in my own teams and in others.

Let's look again at the tasks your team must complete during the contest.

1. Research
2. Come up with strategies to:
  - Model the system.
  - Solve the problem.
  - Quantitatively compare your solutions.
3. Implement these strategies on the computer.
4. Have the computer evaluate the results of your strategies, and figure out how to display your data.
5. Write the paper.

The most important of these goals is the last one. A well written paper with so-so models/solutions/data will always trump a so-so paper with well developed models/solutions/data. So how do you make sure to write a good paper? I can give you lots of suggestions and ideas, but the reality is that it's mostly an issue of TIME. If you can sit down as a team with your paper, and spend 15 minutes or so on each paragraph – read it out loud, look for things that aren't completely clear, talk about ways to rearrange it – then you will end up with a top notch paper.

Let's talk about time a little more. When are you likely to waste time? When will you be rushed? Typically, on Friday and Saturday the contest feels very long and it's easy to waste a lot of time. The pressure will grow as Sunday passes way too quickly, and you will be incredibly rushed on Monday afternoon. You end up hurrying through the one task you should spend the most time on!

The roadblock that wastes all your time is usually task #3 – trying to implement your ideas on the computer. My team spent most of Friday, Saturday, and Sunday on that task when I did the submarine problem. When we finally did get something, it was already Monday. Even if we'd got glorious results, by Sunday night it's too late. We did not have time to spend on the paper, which was lousy as a result, and that's the trap: You spend all your time fighting the computer, so that when you do go to write the paper, you don't have enough time to write a good one.

So how do you beat the trap? In a way I was lucky that we beat it when we did the MRI problem – I happened to know exactly how to implement our ideas (thanks to Numerical Recipes), so that we had almost all the data we needed by the end of Saturday. The problem clicked with what I knew, so we had a huge amount of time to throw at the paper. You can't count on being so lucky.

In order to guarantee that you will have lots of time to spend on the most important goal, you need to set up harsh time limits on the other tasks, then stick to them as closely as you can. Make yourself stay up Thursday night and Friday night in order to meet the deadlines, so you don't have to stay up Saturday or Sunday nights – which can be deadly.

To do well, your team needs to make Friday the most productive day of the contest by DEMANDING that things get done. One simple way to do this is to come up with your strategies as soon as possible, and insist that they be working on the computer by midnight Friday. If your computer person doesn't think he/she can get things finished and running by Friday midnight – then you need to make things simpler. This contest is about spending 96 hours on a 96 day problem. Set things up quick and dirty, but get them working! Stay up Friday night and make sure the computer work is DONE before Saturday rolls around. If it's Friday at 6 pm and you're not going to get the things done within the next few hours, then you're trying to do something too complicated. Regroup, do something simpler, but get that the computer working as fast as possible.

The trap is an issue of time, and if you're aware of it, you should be able to beat it. Do whatever it takes to make sure that all the research, the computer work, and the data collection is DONE by Saturday night at the very latest, then get a good night's sleep. Even if it means rushing through things at the beginning, it's worth it to save the time for the paper. If your team can devote all of Sunday to the paper, then you've already defeated the vast majority of the other teams.

## 6 Useful References

There's no way I can just give you a list of all the references you will need during the contest. Any textbooks you've had in a class can be especially useful, because you've worked through a good part of them and know what they hold. Make sure that they're close at hand on Friday, and start with them.

The art of using a library is a whole subject in itself, but I can give you a few tips. Once you've scanned through your own textbooks, make a list of the things you want to research and do things systematically. You'll most likely be looking for textbooks, not current research or conference proceedings. Remember: Keep it simple. Start with the basics. Find the general classification that your problem falls into, and learn as much as you can. Then try to creatively apply that knowledge to the specific problem that the MCM has given you.

All that being said, there are a number of books that do cover a lot of territory and have information that could be applied to many past problems. These books are generally about solving problems, often with examples of computer programs included to make your life easier. If you find any such books that might be useful to others – please e-mail me and we'll add them to the list! (kcline@carroll.edu)

- Numerical Recipes in C: The Art of Scientific Computing, William H. Press  
This is a truly fantastic book. It gives you a start on just about every continuous modeling technique that can be programmed. Interpolation, solving ordinary and partial differential equations, linear algebra, all with the code ready to be typed in. There are editions of the book in other languages to suit your preference.
- Numerical Methods that Work, Forman Acton  
Another really first rate book. Very practical – a great place to start.
- Introduction to Algorithms, Thomas H. Cormen, Leiserson, Rivest
- Numerical Analysis, Richard L Burden, J. Douglas Faires, Albert C. Reynolds
- Matrix Computations, Gene H. Golub, Charles F. Van Loan
- Introduction to Automata Theory, Languages, and Computation, John E. Hopcroft
- Computer Graphics: Principles and Practice, James D. Foley