Microwave Optics: Analysis of a Cubic Lattice with Bragg Scattering

Zachary G. Shermer'18 and Hugh O. Thurman III

Department of Physics & Astronomy, Hampden-Sydney College, Hampden-Sydney, VA 23943

INTRODUCTION

Bragg Scattering is a process that was discovered by William Bragg in the early 1900s. Bragg discovered that X-rays reflect off of crystals differently at different incident angles. He was able to use this information to create what is now known as Bragg's Law, which can be used to measure the interatomic spacing of a crystal using the following equation:

$$m\lambda = 2d\sin(\theta)$$

Where d is the interatomic spacing, λ is the wavelength of the beam being emitted, θ is the angle of incidence, and m is the integer of the order of diffraction.

Bragg Scattering formed the foundation that would later lead to the Davisson-Germer Experiment with electron diffraction. Davisson and Germer were able to use the interatomic spacing of a crystal and show that electrons had wave properties, which proved deBroglie's earlier hypothesis of wave-particle duality. Everything has a wave property, even humans, albeit miniscule. This would prove to be very important in the development of quantum mechanics.

This experiment used a microwave emitter, receiver, and goniometer, set up with the cubic lattice on a turn table at the center of the geometer arm with the receiver and transmitter on either side. The wavelength of the microwave was calculated to be 2.76 cm, whereas the device was said to emit a 2.85 cm wavelength. This resulted in a discrepancy of 3.16%. For the 100th Plane, the spacing between the ball bearings was calculated to be 4.23 cm as compared to the actual value of 3.8 cm, which resulted in a discrepancy of 11%. The 110th Plane resulted in a value of 2.0 cm as compared to the actual value of 2.7cm, which means there was a discrepancy of 25.93%. The most likely source of error in this lab was the receiver, which had been modified from its original form and could cause systematic error. The receiver itself would not display the data correctly, so a multi-meter was attached in order to take measurements.

Theory

The lattice structure of ball bearings used in this experiment is much larger than the crystal structures used in Bragg Scattering, which means that a microwave ray with a wavelength of λ=2.85 cm can be used, since the wavelength is comparable to the spacing of the ball bearings in the cube lattice. To confirm that the microwave emitter was functioning correctly, the equipment was set up to test the frequency of the microwave being emitted. The following relationship was used:

$$\text{velocity} = \lambda v$$

Where the velocity is the velocity due to air propagation ($3.0 * 10^8 \frac{\text{m}}{\text{sec}}$), λ is the wavelength, and v is the expected frequency for microwave radiation (-10.25 GHz).

The "atomic" planes 100 and 110 refer to the incident angle on the lattice with respect to the microwave emitter, and can then be measured. The equation for Bragg's Law can then be used to calculate the distance between the ball bearings in the crystal lattice.

Methods and Materials

For the first part of the experiment, which is to measure the wavelength of the microwave, the following equipment was needed:

- Transmitter
- Receiver
- Component Holder (2) Goniometer
- Microwave Detector Probe (ME-9319

The purpose of this part of the experiment was to confirm that the microwave emitter was emitting a wave with a wavelength of 2.85cm. In order to do this, the goniometer arm was set up to be straight; that is, the receiver and transmitter were pointed directly at one another. The receiver can was then slid down the goniometer arm to find where the maximum amplitude occurred, which was then recorded. The maximum amplitude occurs where the distance between the transmitter and receiver is equal to $n\lambda/2$. This means that the distance beteen maximum wavelengths is λ/2. Once ten maxima had been found, the distance between the two consecutive maxima could then be found and multiplied by 2.0 to produce the wavelength of the microwave. The same process could be done with the minima. The average of the wavelengths calculated can then be compared to the given wavelength of λ=2.85cm.

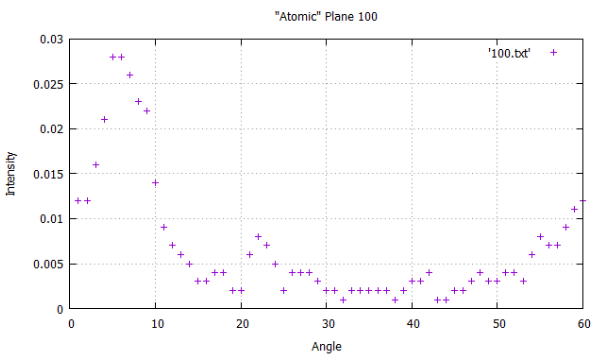
The second part of the lab is the portion of the lab in which the spacing between the ball bearings is measured. For this part of the lab, the following equipment will be needed:

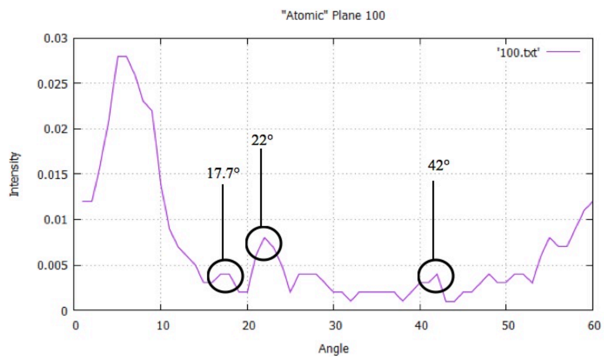
- Transmitter
- Goniometer
- Cubic Lattice
- Receiver
- Rotating Table

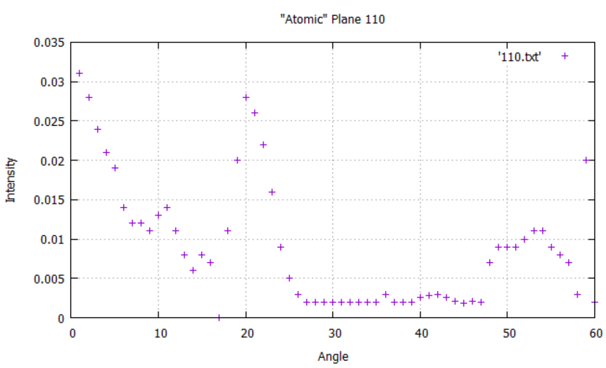
The setup for this portion of the lab is very similar to the first part. The only alteration was addition of the cubic lattice and the rotating table, which sits on the middle of the goniometer. However, it is important to orient the cubic lattice correctly for the “Atomic” Plane that is being measured, which in this case will be the 100th “Atomic” Plane and the 110th “Atomic” Plane. The goniometer should start at 0°. The crystal can then be rotated one degree clockwise while the rotatable goniometer arm is rotated two degrees clockwise. The meter reading should be recorded along with the grazing angle, which will be all the way from 1° to 60° for both “Atomic” Planes. Once the data is taken, the relative intensity should be graphed against the grazing angle.

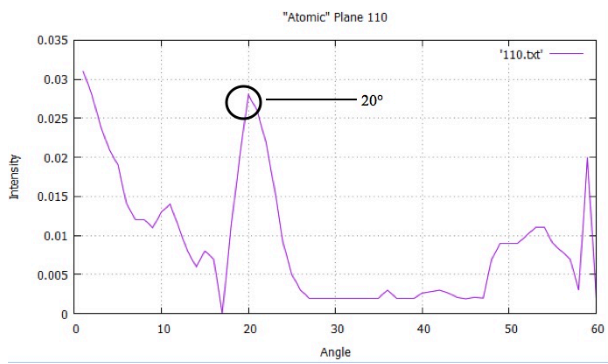
For the 100th “Atomic” Plane there should be several peaks on the graph. Bragg’s equation can then be used to solve for the spacing using the angle the peak occurred at to solve for the distance between the ball bearings in the cube. The actual spacing is 3.8 cm, so percent discrepancy can be calculated from this value. There should be one main peak on the 110th “Atomic” Plane, which correlates directly with the spacing between the ball bearings. The actual spacing for this portion is 2.7cm, so the peak should occur somewhere around 27°.

DATA & FIGURES









Wavelength of the Microwave

Minima (cm)	Intensity (mA)	n-2n, 2n-3n, ... (cm)
51	0	
49.5	0	3
47.85	0	3.3
46.3	0	3.1
44.85	0	2.9
43.7	0	2.3
42.33	0	2.74
41.21	0	2.24
39.95	0	2.52
38.58	0	2.74
	Average:	2.76
	Percent Discrepancy:	3.16%

I chose to use the minima to record the wavelength rather than the maxima, because it seemed more efficient to make the meter zero out than try to find the maxima

Atomic Plane (100)		Atomic Plane (110)	
Degree	Voltage (V)	Degree	Voltage (V)
1	0.012	1	0.031
2	0.012	2	0.028
3	0.016	3	0.024
4	0.021	4	0.021
5	0.028	5	0.019
6	0.028	6	0.014
7	0.026	7	0.012
8	0.023	8	0.012
9	0.022	9	0.011
10	0.014	10	0.013
11	0.009	11	0.014
12	0.007	12	0.011
13	0.006	13	0.008
14	0.005	14	0.006
15	0.003	15	0.008
16	0.003	16	0.007
17	0.004	17	0
18	0.004	18	0.011
19	0.002	19	0.02
20	0.002	20	0.028
21	0.006	21	0.026
22	0.008	22	0.022
23	0.007	23	0.016
24	0.005	24	0.009
25	0.002	25	0.005
26	0.004	26	0.003
27	0.004	27	0.002
28	0.004	28	0.002
29	0.003	29	0.002
30	0.002	30	0.002
31	0.002	31	0.002
32	0.001	32	0.002
33	0.002	33	0.002
34	0.002	34	0.002
35	0.002	35	0.002
36	0.002	36	0.003
37	0.002	37	0.002
38	0.001	38	0.002
39	0.002	39	0.002
40	0.003	40	0.0026
41	0.003	41	0.0028

ANALYSIS & DISCUSSION

After The first part of the lab yielded a wavelength of 2.76 cm. The emitter is labeled as emitting a wavelength of 2.85 cm, creating a discrepancy of 3.16%. While this is minimal, it will compound the error in the second part of the experiment if the emitter is not emitting a microwave with a wavelength of 2.85 cm since the wavelength will be used in the calculations. An important issue to note is that the receiver was tampered with. The receiver was supposed to be battery operated, but it was modified to use a DC current adapter. This change is most likely why the meter on the receiver itself did not work, and a multi-meter had to be used.

The second part of the lab yielded a little more error than the first part. For the 100th Plane, the equation $m\lambda = 2d\sin(\theta)$ has to be used to solve for the spacing between the ball bearings. The three peaks on the graph were used for theta in the equation. M was equal to 1.0 for each point except the 42° peak, where it was equal to 2.0. When the spacing was solved for and averaged out, a spacing of 4.23 cm was found. This resulted in an 11% error when compared to the actual spacing of 3.8 cm. Interestingly enough, if 2.76 cm is used for lambda, the spacing is calculated to be 4.11 cm with a percent discrepancy of 8.16%. This makes sense, seeing as the wavelength of the microwave was measured to be lower than what it is supposed to be. The 110th Plane was significantly off when compared to the 100th Plane. The peak on the graph of the 110th Plane was at 20°, meaning that the spacing should be 2.0 cm. However, the spacing is actually 2.7 cm, a discrepancy of 25.93%.

As stated above, a probable cause of error was the equipment itself. The receiver had been modified from its original condition, which could have been the cause of some error. The receiver also did not display the intensity correctly, so there was a malfunction somewhere in the receiver. Thankfully, a multi-meter provided adequate data collection. This would all be systematic error. As noted in the literature for the experiment, tabletop reflections can interfere with the signal, which could explain why a lower wavelength was calculated. This would be random error, as the lights in the lab tend to flicker. The manual also says to use a clean, flat table for the experiment. The table I used was not perfectly level, which also most likely resulted in systematic error as well.

CONCLUSION

This lab was conducted to measure the spacing between ball bearings in a cubic lattice. This experiment, while scaled up in size to make the data collection easier, utilized Bragg Scattering to calculate the spacing between the ball bearings. In order to do this a goniometer, receiver, microwave transmitter, and they cubic lattice were set up at two different planes (the 100th and 110th) and the spacing between the ball bearings was calculated using Bragg's Law.

The wavelength of the microwave was calculated to be 2.76 cm, which resulted in a discrepancy of 3.16%. The spacing on the 100th Plane was measured to be 4.23 cm as compared to the actual value of 3.8cm, which resulted in a discrepancy of 8.16%. The 110 Plane had its peak at 20°, meaning that the spacing should be 2.0cm. The actual value was 2.7cm, which resulted in a discrepancy of 25.93%.

To improve upon this lab in future experimentations, it would be best to use a better receiver. The receiver that came with the kit did not fully function and had been altered. This almost definitely resulted in systematic error. The goniometer are was also somewhat hard to move without moving the entire setup. The best way to fix this might be to work on a surface on which it can move more easily. The 210 Plane could also be measured in future experiments, although that one is probably the hardest one to line the cubic lattice up for properly.

REFERENCES

1) Bragg Diffraction Experiment. (n.d.). Retrieved December 6, 2015, from http://www.vicphysics.org/documents/teachers/Bragg_Diffraction_Experiment.pdf

2) Instruction Manual and Experiment Guide for the PASCO scientific Model WA-9314B (Vol. Microwave Optics, pp. 1-5, 13-15, 33-34, 44). (n.d.). Roseville, CA: PASCO.

3) **Kenneth Krane**. *Modern Physics Third Edition*. New York, NY : John Wiley and Sons, INC, 2012.