

Determining the Kinetic Coefficient of Ice Using a Tribometer

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Skiing has been around for over 4,000 years [8]. The sport has made great progress since then. We now are starting to really understand how to improve the sport and the speed of skis by using physics and physical analysis. One of the main components of skiing that people need to understand is the interaction between the snow, and the snow ski. There is a bit more to understand than friction, because you have to look at the components that make up the frictional force. The components that make up the frictional force are the ones studied in this experiment. Adhesion, capillary bonding, contact angle, and surface roughness all play an important part in the interaction between the snow and the snow ski, which affect the coefficient of kinetic friction. This experiment is designed to measure the coefficient of kinetic friction of snow skis, both waxed and un-waxed in a lab setting.

Introduction

Theory Surface and Coatings Technology: A solid surface with "vee" grooves representing a roughened polyethylene ski attached to a flat ice surface. Snow surfaces consist of a series of these flattened ice surfaces, and the problem analyzed here assumes that the wavelength of the surface roughness is much smaller than the length of the flat ice grains. Since the lower surface is ice, I also assume that the contact angle on that surface is 0° whereas the contact angle of water on the polyethylene surface is a parameter, θ . We could easily generalize the result to a flat surface that is not wetted completely.

The polyethylene consists of a flat section of length $2L$ and a groove of angle α . For most water pressures, the flat solid is not completely wetted and thus air bubbles, which are assumed to be at atmospheric pressure, occupy part of the contact area. We assume that the polyethylene is not in direct contact with the ice everywhere, but that there is a statistical distribution of the separation distances, H .

We are primarily interested in finding two things: the contact area and the force, which is the area times the tension. The area is available from

$$R = x_c + [r^2 - (r - y_c)^2]^{1/2},$$

but we take the tension as a parameter since there is no way to solve for the tension independently. For grooves of length l the area, A , is given by

$$A = 2l \left(L + \frac{r-H}{\tan \alpha} \right) + 2lr \frac{\cos \theta}{\sin \alpha},$$

and force, F , is given by

$$F = 2lT \left(L + \frac{r-H}{\tan \alpha} \right) + 2l\sigma \frac{\cos \theta}{\sin \alpha}.$$

For a given tension and solid geometry, this immediately suggests that the adhesion decreases as the contact angle increases through the entire range

of possible values. This agrees with skiing experience where softer, more hydrophobic waxes are used to repel water at higher temperatures. It suggests that, other things being constant, adhesion decreases most rapidly around a θ of 90° , which is in the range of both polyethylene and paraffin waxes. It also suggests that the new synthetic fluoro-waxes are successful because a small change in contact angle beyond 90° can be significant. For example, a change of 10° from 90° to 100° gives a 17% decrease in $\cos \theta$.

Experimental setup for observing the area of wetted contact through a microscope slide adhering to a ceramic plate by water tension. The wetted areas of glass microscope slides were determined visually by placing them against porous plates of ceramic and varying the tension in the water film. The tension was measured, while moving the water reservoir changed the height of the water column. The wetted area was observed visually and computed with an image analysis system. The wetted area remained at 100% as tension increased to T_1 in each case and, except for the smoothest surface, it eventually approached zero. The actual setup involves a tube, funnel, glass plate, and ceramic plate. Adjusting the tube down from level with the porous ceramic plate creates the tension. [6]

Theory of Capillary Bonding: Capillary bonding occurs because the water film is in a state of 'tension', meaning that its pressure is below atmospheric pressure (i.e. tension equals atmospheric pressure minus absolute water pressure; it is measured as cm of H₂O but is actually a pressure). Altering the amount of water present, the surface chemistry, or the surface roughness in ways that are often exploited but poorly understood can change capillary bonding. These effects were partly explained by a simple model of capillary bonding forces and some direct observations of the capillary bonding of glass to porous ceramic. For all of these materials, the 'fractional wetted area' (i.e. the fraction of the total area which is wetted and through which

the capillary bonding occurs; it is calculated as force divided by tension and total area) decreased as the tension in the water film increased, but the capillary bonding force reached a maximum at an intermediate value of tension. This maximum value of capillary bonding occurs at an intermediate value of tension because capillary bonding is the product of the decreasing 'wetted area' (i.e. the total area of a sample which is wetted) times the increasing tension.

Making these measurements along with simultaneous observations of wetted area using glass or ceramic tested this technique. This was necessary to develop an understanding of the technique's strengths and pitfalls. Once developed, the technique was applied to roughened glass, aluminum, and polyethylene, to explore the effects of roughness, and applied to polyethylene and glass coated with various ski waxes, to explore the effects of contact angle. These results show how the technique can be successfully applied to measure capillary bonding, although the procedure will produce inconsistent and meaningless results unless used very carefully. The need for careful use of this technique cannot be overemphasized.

A direct measurement of the release force was added to the experimental set-up so that the capillary bonding properties of materials other than glass could be investigated. Force was applied at an increasing rate using a motor drive attached to a transducer. The transducer was lifted vertically at a constant speed until the test plate released from the ceramic. The wetted area and capillary bonding force were measured simultaneously for glass on ceramic to evaluate the new procedure. The wetted area was observed directly through the glass, although its measurement was not precise because a portion of the water film under the glass plate was hidden by the connection to the force transducer, which was attached above the center of the glass plate.

Parts used in experimental setup, string, force transducer, motor driven screw, glass, aluminum, or polyethylene plate, water film, ceramic plate, water, tension, tube. Layout of the device used to make the force measurements. A motor turned a screw drive, which lifted the force transducer, which was attached to the plate by a thread. [3]

Theory

Theory of measuring the coefficient of kinetic friction is pretty simple.

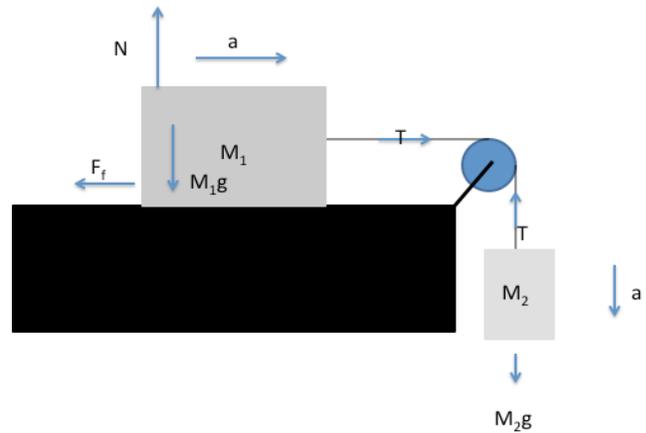


Figure 1: This force diagram illustrates the forces involve with a hanging masses attached to another mass across a pulley.

The summation of the free body diagram comes up with the equations:

$$(1) \quad T - M_2g = -M_2a$$

$$(2) \quad T - F_f = M_1a.$$

T is tension, F_f is the frictional force, g is the gravitational constant, and a is the acceleration. From these two equations you get the equation:

$$(3) \quad T = M_2g - M_2a.$$

Next you can substitute in and form the equation:

$$(4) \quad M_2g - M_2a - F_f = M_1a$$

Solving for the acceleration (a) we come up with the following equations:

$$(5) \quad \mu_k = \frac{M_2}{M_1}$$

Where μ_k is the coefficient of kinetic friction. The second equation is:

$$(6) \quad \mu_k = \frac{M_2g - (M_1 + M_2)a}{M_1g}$$

We use the second equation if the acceleration is not close to zero, because it would mess up our results otherwise. This does account for the friction of the pulley, or the moment of inertia of the pulley.

Methods and Experimentation

The materials required for this experiment are the sample of base material from a snow ski, various ski waxes, a string, a scale, a pulley, an instrument that you are able to put various weight on for the hanging mass, a motion sensor, and a way to read the motion sensors data. Place the ski base material sample on the ice surface, and then attach the string to the base material. Next, run the piece of string over the pulley and place the hanging mass on the string. It is important to make sure that the ski and ice surface are level, and that the string is level with the pulley so there is no added resistance. It is also important that the string stays taught throughout the data recording process, so that the acceleration measured by the motion sensor is accurate. Add weight to the hanging mass until the base material slides across the ice surface. Once the desired weight of the hanging mass is found, record the acceleration of the system (this should be the liner slope of the velocity graph measured by the motion sensor). Weigh the hanging mass and record the weight, and then weigh the mass of the base material and record its weight. Knowing these two weights, and the force vectors calculate the coefficient of kinetic friction using the formula (6):

$$\mu_k = \frac{M_2g - (M_1 + M_2)a}{M_1g}$$

We repeat this process for all of the base samples.

Data

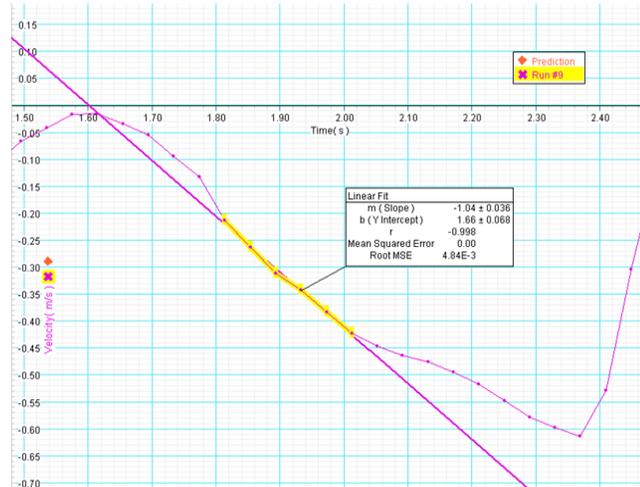


Figure 2: Above is a sample run for the Un-waxed Control Block. We record the velocity of the block with respect to time. By analyzing the slope, we determine the accelerations that we use to calculate the coefficients of kinetic friction in each run.

| M1 (g) | M2 (g) | Acceleration (m/s ²) | Coefficient of Friction | Number of data points | $\Delta\mu$ | Δa |
|-------------------|--------|----------------------------------|-------------------------|-----------------------|-------------|------------|
| 108.2 | 37.13 | 1.01 | 0.204874124 | 2 | 0.0001203 | 0 |
| | 37.13 | 1.05 | 0.199397424 | 14 | 0.00188084 | 9.60E-03 |
| | 37.13 | 0.912 | 0.21829204 | 2 | 0.00012818 | 0 |
| | 37.13 | 0.925 | 0.216512113 | 3 | 0.00237984 | 9.90E-03 |
| | 37.13 | 1.04 | 0.200766599 | 4 | 0.00797224 | 0.041 |
| | 37.13 | 1.17 | 0.182967322 | 18 | 0.01318833 | 0.084 |
| | 37.13 | 1.11 | 0.191182373 | 6 | 0.00281083 | 0.016 |
| | 37.13 | 0.995 | 0.206927887 | 3 | 0.03333382 | 0.16 |
| | 37.13 | 1.04 | 0.200766599 | 6 | 0.00700704 | 0.036 |
| | 37.13 | 0.905 | 0.219250463 | 3 | 0.00012874 | 2.40E-14 |
| | 37.13 | 1.03 | 0.202135774 | 5 | 0.00928143 | 0.047 |
| Average | | | 0.203915701 | | 0.00711196 | |
| AVG Data pts 4-18 | | | 0.196202682 | | 0.0421407 | |

Table 1: The table above shows the data gathered for the Unwaxed Control Block. We record the two masses, the trials' acceleration, the number of data points used to approximate each acceleration value, the calculated coefficients, and the changes in acceleration and the coefficients.

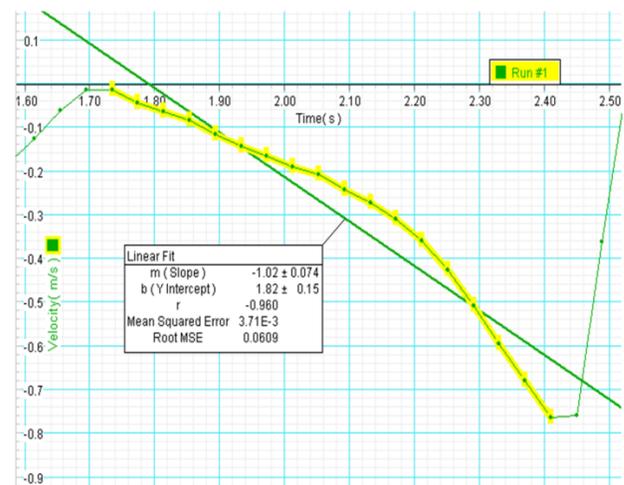


Figure 3: Above is a sample run using Swix Yellow Race Wax (28-50°F).

| M1 (g) | M2 (g) | Acceleration (m/s ²) | Δa | Coefficient of Friction (μ) | $\Delta\mu$ |
|---------|--------|----------------------------------|------------|-----------------------------------|-------------|
| 142 | 56.99 | 1.02 | 0.074 | 0.255633157 | 0.01873205 |
| | 56.99 | 1.03 | 0.042 | 0.254204678 | 0.01055117 |
| | 56.99 | 1.26 | 0.048 | 0.221349658 | 0.00859399 |
| | 56.99 | 1.09 | 0.032 | 0.245633803 | 0.00739095 |
| | 56.99 | 1.39 | 0.050 | 0.202779429 | 0.00744234 |
| | 56.99 | 0.987 | 0.059 | 0.260347138 | 0.01575248 |
| | 56.99 | 0.749 | 0.075 | 0.294344941 | 0.02968791 |
| | 56.99 | 0.737 | 0.080 | 0.296059116 | 0.032352 |
| | 56.99 | 0.688 | 0.053 | 0.303058664 | 0.0235667 |
| | 56.99 | 0.647 | 0.064 | 0.308915428 | 0.03078204 |
| | 56.99 | 0.665 | 0.014 | 0.306344166 | 0.00667413 |
| | 56.99 | 0.849 | 0.015 | 0.28006015 | 0.00515398 |
| Average | | | | 0.269060861 | 0.01638998 |

Table 2: The table above shows the data gathered from using Swix Yellow Race Wax (28-50°F).

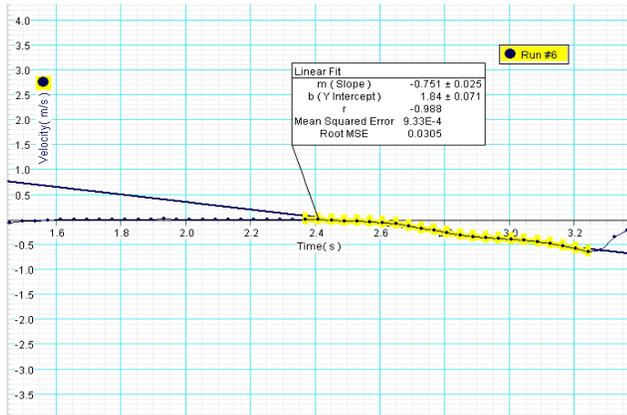


Figure 4: Above is a sample run using Swix Red Race Wax (-12-36°F).

| M1 (g) | M2 (g) | Acceleration (m/s ²) | Δa | Coefficient of Friction (μ) | Δμ |
|---------|--------|----------------------------------|-------|-----------------------------|------------|
| 141 | 35.16 | 0.756 | 0.044 | 0.153080617 | 0.00934559 |
| | 35.16 | 0.623 | 0.026 | 0.170018956 | 0.00758016 |
| | 35.16 | 0.724 | 0.056 | 0.157156007 | 0.0126033 |
| | 35.16 | 0.768 | 0.064 | 0.151552346 | 0.01306096 |
| | 35.16 | 0.767 | 0.082 | 0.151679702 | 0.01664795 |
| | 35.16 | 0.751 | 0.025 | 0.153717396 | 0.00555551 |
| | 35.16 | 0.775 | 0.062 | 0.150660854 | 0.01248195 |
| | 35.16 | 0.780 | 0.066 | 0.150024074 | 0.01312158 |
| | 35.16 | 0.734 | 0.023 | 0.155882447 | 0.00532926 |
| | 35.16 | 0.747 | 0.046 | 0.15422682 | 0.0099366 |
| | 35.16 | 0.760 | 0.049 | 0.152571193 | 0.01027145 |
| | 35.16 | 0.761 | 0.058 | 0.152443837 | 0.01205276 |
| Average | | | | 0.154417854 | 0.01066559 |

Table 3: The table above shows the data gathered from using Swix Red Race Wax (-12-36°F).

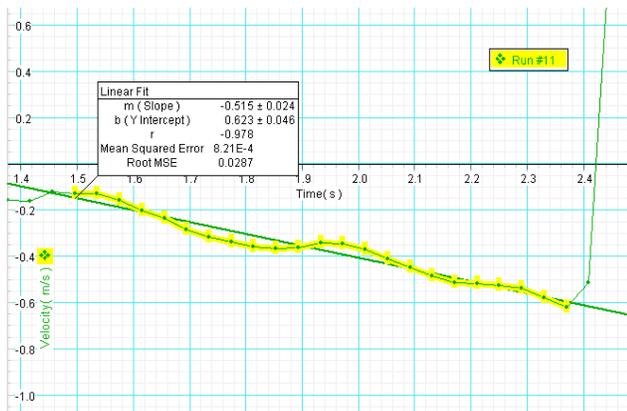


Figure 5: Above is a sample run using One Ball Jay All Temperature Wax.

| M1 (g) | M2 (g) | Acceleration (m/s ²) | Δa | Coefficient of Friction (μ) | Δμ |
|---------|--------|----------------------------------|-------|-----------------------------|------------|
| 143 | 50.53 | 0.701 | 0.042 | 0.256648895 | 0.01556427 |
| | 50.53 | 0.565 | 0.061 | 0.275411026 | 0.02993526 |
| | 50.53 | 0.605 | 0.036 | 0.269892753 | 0.0162567 |
| | 50.53 | 0.715 | 0.034 | 0.2547175 | 0.01229853 |
| | 50.53 | 0.506 | 0.054 | 0.28355048 | 0.03046688 |
| | 50.53 | 0.624 | 0.033 | 0.267271572 | 0.01432971 |
| | 50.53 | 0.561 | 0.049 | 0.275962854 | 0.02430484 |
| | 50.53 | 0.518 | 0.060 | 0.281894998 | 0.03285724 |
| | 50.53 | 0.563 | 0.042 | 0.27568694 | 0.02076737 |
| | 50.53 | 0.567 | 0.030 | 0.275135113 | 0.01475831 |
| | 50.53 | 0.515 | 0.024 | 0.282308869 | 0.01336241 |
| | 50.53 | 0.558 | 0.070 | 0.276376724 | 0.03487218 |
| Average | | | | 0.27290481 | 0.02164781 |

Table 4: The table above shows the data gathered from using One Ball Jay All Temperature Wax.

| Block Used | Coefficient of Friction (μ) | Δμ |
|----------------------------------|-----------------------------|--------|
| Unwaxed Control | 0.1962 | 0.0421 |
| Swix Yellow Race Wax (28-50°F) | 0.2691 | 0.0164 |
| Swix Red Race Wax (-12-36°F) | 0.1544 | 0.0107 |
| One Ball Jay All Temperature Wax | 0.2729 | 0.0216 |

Table 5: Above is a table comparing of the average coefficients of kinetic friction and the average changes in the coefficients for each system.

ANALYSIS

For the un-waxed control block I came up with the coefficient of kinetic friction to be 0.1962 with an uncertainty value of plus or minus 0.0421. The Swix yellow race wax had a coefficient of kinetic friction of 0.2691 with an uncertainty of plus or minus 0.0164. The Swix red race wax had a coefficient of kinetic friction of 0.1544 with an uncertainty of plus or minus 0.0107. The One Ball Jay all temp wax had a coefficient of kinetic friction of 0.2729 with an uncertainty of plus or minus 0.0216. The Swix red race wax had the lowest coefficient of kinetic friction. The un-waxed control block had the second lowest coefficient of friction. The Swix yellow race wax had the third lowest coefficient of friction, and the One Ball Jay all temperature wax had the highest coefficient of friction. There may have been systematic errors in this experiment with not having a constant temperature reading of the ice that the sample slid across, my not reading the graph of the motion sensor correctly, or error in the transposition of numbers from the graph to my data table. Random errors include the motion sensor only being so accurate, and the motion sensor could have dropped some of the data points.

CONCLUSION

The Swix red race wax had the lowest coefficient of kinetic friction. The Swix red race wax

had a coefficient of kinetic friction of 0.1544 with an uncertainty of plus or minus 0.0107. The coefficient of kinetic friction for the un-waxed control block was 0.1962 with an uncertainty value of plus or minus 0.0421. The Swix yellow race wax had a coefficient of kinetic friction of 0.2691 with an uncertainty of plus or minus 0.0164. The One Ball Jay all temp wax had a coefficient of kinetic friction of 0.2729 with an uncertainty of plus or minus 0.0216. The method that was very simple, which analyzing vector quantities from the free body diagram and then calculating the coefficient of friction based on the vector quantities. For future experiments I should measure the temperature of the ice surface in order to maintain a constant temperature for all of the data runs. To further improve the experiment, get into testing the capillary bonding and adhesion of the base materials.

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