

Electromagnetically Coupled Oscillators

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ABSTRACT

The experiments performed were designed to explore electromagnetically coupled oscillators and energy transfer through various forms. This project is important for later research and experimentation involving resonance in electromagnetically coupled oscillators. Ferromagnets were suspended from springs with similar spring constants inside a coil. The coils were wired together and one magnet was forced to oscillate while the second magnet was observed. The conclusion of the experiment found that roughly 100 mV is required on the system in order to have measurable displacement in the second magnet and that this potential cannot be achieved with the parameters in place on the experiment. In order for displacement to be large enough to measure resonance in later experiments, the potential must be amplified before it is transferred to the second coil.

INTRODUCTION

The goal of this project is to examine electromagnetically coupled oscillators and the properties of work, electromagnetic induction, and energy conservation. The topics explored are electromagnetism, electric generation, and the relationship between mechanical energy and electromagnetic energy. The experiment was designed with two coils coupled through wires with a magnet suspended on a spring in each coil. The concept is that when the magnet in one coil begins to oscillate, it creates an induced electromagnetic field (emf) on the wire which is transferred to the adjacent coil, causing oscillation to occur for the second magnet as well. The most important variable is the efficiency of the system because any potential energy created by the motion of the first magnet is propagated to the second at a fractional rate. This research is intended to supplement information about future research projects and how to achieve resonance using an external electromagnetic force.

The history of this project is complex because of the various fields of study involved. Originally, electricity and magnetism were not related, but were considered two completely separate forces. It wasn't until James Maxwell published his work relating electricity and magnetism in 1873 that the relationship was established. Both electricity and magnetism are directly related to the repulsion and attraction of subatomic particles: in particular, the electron. The electron was discovered by J. J. Thompson in 1897. Thompson studied cathode rays being emitted

through an electromagnetic field. "He observed that the cathode rays were deflected by both electric and magnetic fields - they were obviously electrically charged^[1] and that he was able to derive a ratio of the charge of the electron to its mass. It wasn't until 1909 that Robert Millikan would determine the exact mass of the electron.

Electrons are important because they are the medium through which electricity transfers. In this experiment, an electromagnet is used in the form of a coil to generate electricity through a changing magnetic field. When a magnetic field acts on a coil, the electrons in the coil are forced in the direction opposite to the field. The movement of these electrons creates a voltage potential and when connected in a circuit, current can flow. This process can also be done reversibly. When electricity moves through a coil, a magnetic field is produced and this magnetic field can act on objects within the field, such as a mass suspended on a spring.

In this experiment, "the system is mechanical but its coupling is electromagnetic²" and that adds to the complexity of the concepts and calculations. The magnetic force has to tie in with the force of a spring, Hooke's Law. Hooke's law is the physical principle governing harmonic motion for a spring.

THEORY

In order to understand electromagnetic energy transfer it is important to understand electromagnetic force. The equation for electromagnetic force is

$$(1) \quad \vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

which shows that electric force and magnetic force are related. The first part of the equation represents electric force, which is simply in the direction of the electric field. The magnetic force is expressed by the second half of the equation where q is the charge, v is the velocity of the charge, and B is the magnetic field.

When looking at an electromagnetic field acting on a coil, Faraday's Law of Induction is used, expressed as

$$(2) \quad \varepsilon = -N \frac{d\phi_B}{dt}$$

where ε is the electromotive force (V), N is the number of turns in the coil, and ϕ_B is magnetic flux. This equation is a more accurate reconfiguration of the electromagnetic force equation because it accounts for the magnetic force acting uniformly on

the entire surface of the coil and also converts it to voltage. The magnetic flux can also be rewritten as

$$(3) \quad d\phi_B = B \cdot dA \text{ or } Bw \cdot dl$$

and therefore voltage can be rewritten as

$$(4) \quad \varepsilon = -N(B \cdot dA)$$

It is important to also remember the force of gravity, with the equation

$$(5) \quad F = mg$$

When a spring is at rest, the force of the spring, represented as

$$(6) \quad F_{spring} = -\frac{1}{2}kx$$

is equal to the force of gravity. The equation with these two forces equal to each other can be used to find the spring constant, k .

We know that "a magnet in relative motion to a surrounding coil, produces an induced voltage (emf) across it, whose magnitude can be shown to be proportional to the instantaneous velocity of the magnet³." The proportionality can be found in final derived equation

$$(7) \quad \frac{k}{2Bw_2} x_2 = v_1$$

The equation is used to calculate the velocity of the magnet needed to create enough potential to accelerate the second magnet, and is found by setting the force of the spring equal to the electromotive force. In this equation, x is the displacement of the second spring and v is the velocity of the first spring. The equation does not account for inefficiencies in the system, however, so a constant is needed to account for the inefficiency.

EXPERIMENTAL PROCEDURE

We use two 560-turn coils, a variable voltage supply, and ten ferromagnets. We use four banana clips, two hangers, 2 lead weights, and a banana clip-oscillator adapter. An oscilloscope provides us with visual measurements. The magnetic field strength of one ferromagnet is theoretically 10.6 G or 10.6×10^{-4} T at .04 m, according to the supplier website. The Voltage Source has a variable voltage from 0-20 V, as well as a constant +5 V voltage. We use springs with the corresponding spring constants: 10.2617801 N/m, 12.09876543 N/m, 12.17391304 N/m, 9.245283019 N/m, and 9.201877934 N/m.

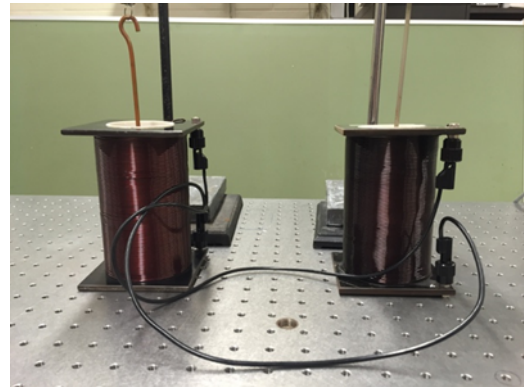


Figure 1: The image shows the setup for the first oscillation experiment.

The first experiment was a purely observational test to see if the two coupled magnets could affect each other in oscillation. The two magnets were suspended from the hangers within each coil. The coils were then connected using two wires and one of the magnets underwent forced spring harmonics. No observable affect was seen on the other magnet. Magnetic field strength was increased (in number of magnets) and velocity was also increased, yet the same results occurred.

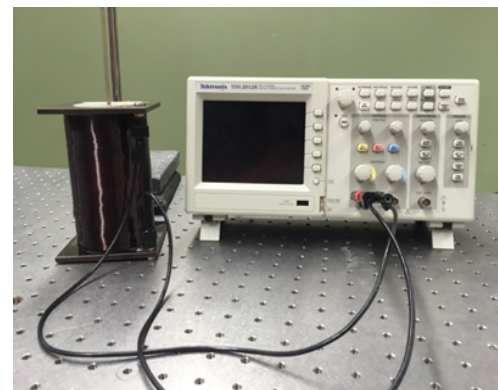


Figure 2: The image shows the setup for the second oscillation experiment.

The second experiment was set out to obtain the base values of V that the magnet could reasonably produce. This experiment was performed by connecting one of the coils to the oscilloscope in order to read the potential that the magnet produced. The base value is taken when the magnet is undergoing zero movement, and this value is used as a comparison to the other values. The magnet was then oscillated by hand to produce a signal. This signal can then be used as a base for the amplification needed to accelerate the second magnet.

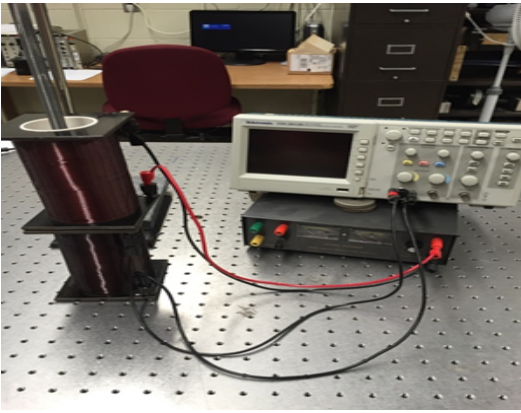


Figure 3: This is a display of third setup of the coils.

The third experiment looked at the force required from the opposite view of the last two experiments. The minimum change in voltage over time needed to accelerate a magnet was found by applying a force over the magnet using a voltage source. The voltage source was connected to the second coil and placed over top of the first coil in the second experiment set up. The voltage source was then adjusted from 0-5 volts in a sinusoidal pattern and visible oscillation was observed on the magnet.

RESULTS

The results of the first experiment were inconclusive because there was no observable change. When the first magnet was oscillated, any voltage possibly generated was loss due to inefficiency. The second magnet saw no displacement and no obvious forces acted on the system.

The second experiment was designed to calculate the electric potential generated by the magnet in oscillation. The magnet was oscillated at a rate of 3 m/s for 100 oscillations. The displacement was 5 cm and the peak to peak voltage potential ranged from 30 mV to 45 mV.

The third experiment exerted a magnetic force on the system using a sinusoidal wave for the external voltage. Displacement was seen when the change in voltage was 2 volts.

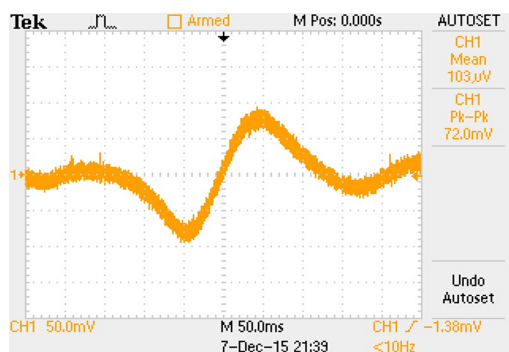


Figure 4: Waveform of the output voltage from the coils during oscillation of the magnet.

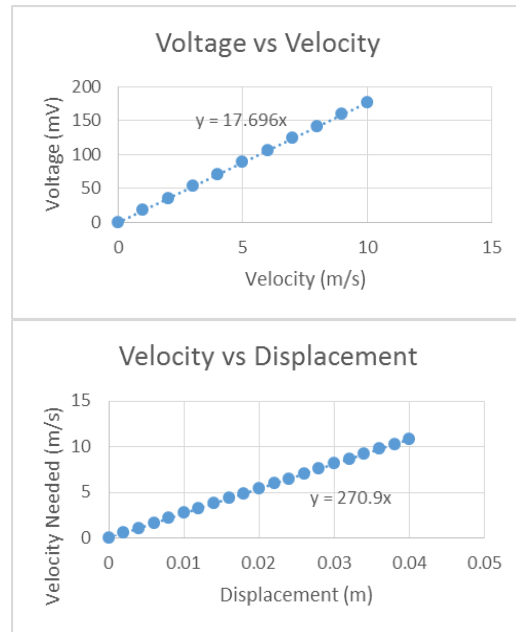


Figure 5: The two graphs illustrate the relationship that velocity has with voltage and displacement.

ANALYSIS

The magnetic strength, B, of the magnets was found to be 152.11 μT per magnet at the radius of the coil. The spring constant was found to be 9.224 N/m and the width of the coil to the magnet was 0.04 m. These values can be used with equation 7 to calculate the velocity needed to displace the spring by an ideal amplitude. When using 5 magnets and an expected displacement of .03 m, the velocity of the first spring is required to be 8.14 m/s. This number is very fast relative to the materials and methods listed here. Because the displacement could not be achieved due to such a high value of velocity, another approach was taken to determine what values of potential are being outputted. The peak to peak voltage found in the second experiment is important for determining the value of voltage for other variables. By calculating the velocity relative to the voltage produced in experiment 2, it is possible to determine the output voltages for other values of magnetic field and velocity. The calculated velocity of the magnet in experiment 2 is 2.94 m/s.

Using the formula in equation 4 and the velocity required above, the output voltage created by the motion of the magnet is 138.57 mV. The voltage produced is the minimum voltage needed to displace the second magnet by .03 m. The voltage required to produce an observable displacement is optimally over 100 mV.

CONCLUSION

Although an observable displacement on the second magnet was never observed, the experiment still provides valuable insight. The purpose of this experiment was to find more information regarding electromagnetically coupled coils, especially how they interact. The results of this experiment show a relationship between both the velocity and displacement and the velocity and voltage potential. These two relationships are important for understanding how the magnets interacted with the coils and how they will interact with each other at higher voltages. The voltage needed to observably displace the second magnet is around 100 mV, a number that can be achievable through amplification using methods such as Op-Amps and transistors. The velocity calculated for the same displacement was 8.14 m/s, however, this value is not as crucial because a lower velocity can be used with amplification methods.

The third experiment was considered a failure because it could not accurately represent the voltage needed to displace the second magnet. This failure is attribute to several factors. The largest factor is that the voltage source was not very accurate; 2 volts was the most accurate measurement of the voltage needed which is not precise enough for the application. It is also inaccurate because this voltage is applied to another coil placed over the magnet. This coil is not going to act upon the magnet in the same way that the corresponding coil would. Lastly, the electromagnetic field that the forcing coil exerts not only affects the magnet but also affects the magnetic field which adds another variable to the equation that cannot be controlled. This experiment can be used as a conceptual representation of a forcing-function acting on the spring.

The values determined in this project will be considered when developing a procedure for the next project, which deals with resonance and viscosity of fluids. This project was largely based on mathematic calculations and experimental observations. The relationship between mechanical and electromagnetic energy is not easily connected. Furthermore, electromagnetic fields are susceptible to large error due to their tendency to affect surrounding systems and the principle that magnetic field strength decreases exponentially over a distance. One potential problem is keeping motion in the z-plane. Any "small initial perturbations rapidly develop into transverse coupled oscillations²" and the force is no longer linear. This project could be improved upon by the use of stronger magnets and a longer coil. These two modifications would allow the magnet to oscillate more freely as well as increase the magnitude of the voltage created by the oscillation.

REFERENCES

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