Measuring phase speed at the single bubble resonance through a bubbly liquid using the standard transfer function technique

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INTRODUCTION

Because sound is a longitudinal wave, it requires a medium to propagate through. Sound speed through a given medium is dependent on the properties of that medium, such as density and temperature. Measuring the sound speed in simple mediums, such as air and water, is easy. Direct measurements can be taken with intuitive time-of-flight style calculations, allowing observers to easily deduce the speed of sound; however, more complex mediums require a more complex method for solving this problem. In this experiment, a bubbly water mixture is observed. By studying the acoustic properties of bubbly water, researchers help to uncover some useful knowledge that can be applied to many fields. For example, in Preston Scott Wilson's dissertation on sound speeds through bubbly liquids, he talks about the importance that this study has on a boat's sonar sensitivity. When in the shallow depths, boats have a hard time mapping out the ocean floor because of the amount of bubbles in the water. Sound is slowed down and dispersed as it enters the new, bubbly medium.[1] If theorists can better predict how sound at certain frequencies will behave when travelling through this bubbly water, observing the ocean floor at shallow depths will be much easier for those who rely on the accuracy of this imaging technique.

Over the last sixty years, researchers have worked to determine the most efficient and accurate method to make these measurements with the commonly used impedance tube approach. In this process, sound is emitted into an isolated tube of known properties and comes in contact with the material being observed. Over time, researchers have worked to improve the impedance tube method and have made it possible to observe the total sound input and sound output of complex mediums, and by using the calculated absorption coefficient, determine the acoustic properties of these different mediums being looked at. This experiment is focusing on the observation of sound speed through a bubbly water medium.

In this experiment, two techniques were used to measure the sound absorption of this bubbly mixture, both using a broadband sound source to drive the bubbles in the medium. These methods are called the one and two microphone transfer function techniques. A method that is commonly presented with these two is the standing wave ratio method. Although this will be tested in future experiments, the two different transfer function methods are being addressed in this paper. We are presenting the results for each method and their corresponding experimental setup, aiming to provide concrete data sets that accurately describe the phase speed of a bubbly liquid, depending on sound frequency and bubble size.

Theory

The theory that this experiment follows is that of the bubble resonance theory, introduced by Commander and Prospertetti. It mathematically describes the behavior of a bubble when exposed to some frequency of sound. The equation that is formulated by this theory is given below:[2]

$$\frac{c^2}{c_m^2} = \frac{1 + 4\pi c^2 na}{(w_o^2 - w^2 + 2ibw)}$$

In order to measure the phase speed of a bubble, it must be indirectly measured by looking at a collective body of bubbles. If a bubbly mixture is treated as a uniform medium for a sound wave to propagate through, the properties of the bubble that makes up this medium can be determined. By sending sound into this medium and observing how it behaves, the bubbly medium's properties can be deduced. The main concept that this experiment's procedure is dependent on is the reflection coefficient, or the ability to measure a sound wave's reflection as it enters a new medium. This states that when a sound wave enters this new medium, a fraction of that wave is reflected back into the initial medium. The amount of sound reflected is dependent on the density of the new medium. This concept can be directly observed when someone shouts at a brick wall. The wall is very dense, reflecting almost all of the sound; however, if another person listens carefully, he or she can observe a small amount of sound travelling through the wall. In this experiment, two different medium interfaces were used. A water to bubbly water interface and an air to bubbly water interface was used.

In order to fully understand how this medium interface is going to behave, each detail about it must be understood and controlled. Sound speed through water is simple. It travels at 1500 m/s, give or take some depending on the temperature of the water. When bubbles are introduced into the system, speed of sound becomes much more abstract. It is dependent on the properties of the bubbly medium, such as bubble size and number. When sound sees a bubbly medium, it sees a compressible fluid that can 'cushion' the sound pressure wave that comes into contact with this medium. This compressibility is really the individual oscillations of each bubble in the medium.[3] At different sound frequencies, the bubbles will oscillate faster or slower, depending on the size. Each bubble size has what is called its resonating frequency. This is where the sound pressure wave is driving the bubble in such a way that the bubble is completely out of phase with the pressure wave. At this moment, the pressure wave strikes the bubble at the bubble's maximum expansion on the same movement plane as the wave. It essentially observes a very dense medium. This drops the speed of the sound down to values two orders of magnitude less than in water alone. Bubble size determines where this drop will occur, and the amount of bubbles, or void fraction percentage, in the medium determines how clear the effect will be. Less bubbles gives a very sharp dip in the theory curve, but it much more difficult to observe.[1]

When observing this sound as it travels through the impedance tube, the collectors read the sound frequency in the time domain. This is a problem because the bubble oscillations are dependent on the frequency of sound traveling through them. The mathematics for this experiment is dependent on data being collected in the frequency domain. To make this change, a fast Fourier transform (FFT) must be applied to the data. With a spectrum analyzer, this transform can be applied very easily and quickly. The mathematics being used is based off of the transfer function. Conceptually, this is a simple input-output function that is used to describe the dynamics of a system that some input travels through. This experiment uses it as a system dependent constant that is in terms of sound strength with respect to frequency. Below are two of the many ways to define this function:

$$Transfer\ Function = \frac{FFT(output)}{FFT(input)} \quad \text{(Eq. 1)}$$

$$Transfer\ Function = \frac{p_2 p_1^*}{p_1 p_1^*} \quad \text{(Eq. 2)}$$

Equation 2 is the signal analyzer input that immediately calculates the transfer function in terms of phase and magnitude. This is used in both of the methods for observing the bubble resonance phase speed.

For the faster, one microphone technique, the process involves data collection at multiple positions inside the impedance tube so that there is more averaging in the data analysis. This is necessary because of the varying sound pressure intensity throughout the tube.[3] Because the hydrophone inside the tube is being moved in different locations, a change in the tube's termination length, or top of the water interface, is occurring. This change will alter the resonating frequencies in the tube. Because of this, the equation to find the acoustic impedance of the bubbly liquid is given as:[1]

$$\frac{z}{\varphi c} = \frac{Ay_1 + B}{y_1 - y_o} \quad \text{(Eq. 3)}$$

In this equation, the term y_1 is the transfer function of the white noise sound inside the impedance tube. The terms A, B, and y_o are all parameter constants that are dependent on the tube's geometric properties. The equations are given below, and are modified versions of the parameter method given in Preston Scot Wilson's dissertation with equation 3:

$$y_o = \frac{f_3(y_1 - y_2) - f_1 y_1(y_2 + y_3) + f_2 y_2(y_3 - y_1)}{f_3(y_1 - y_2) - f_1(y_2 + y_3) + f_2(y_3 - y_1)} \quad \text{(Eq. 4)}$$

$$B = \frac{1}{y_1 - y_2}\left(f_1 y_2(y_1 - y_o) + f_2 y_1(y_2 - y_o)\right) \quad \text{(Eq. 5)}$$

$$A = \frac{1}{y_1 - y_2}\left(f_1(y_1 - y_o) - f_2(y_2 - y_1)\right) \quad \text{(Eq. 6)}$$

With the acoustic impedance calculate, the sound speed through the bubbly liquid, at all frequency values of the spectrum, can be found.

For the more precise, two microphone technique, data is collected at one pair of microphone positions, multiple times. Because of the lengthy calibration process for this method, long term experiments observing one singular spot on the tube; however, it is not practical for multiple locations because the calibration is only able to allow for one position configuration at a time. This calibration process is necessary because of the phase difference that is present between the two microphones.[4] To make this difference negligible, a calibration factor needs to be calculated. To do this, the transfer function of a broadband sound spectrum must be taken inside of a singular medium interface. The equation for the calibration factor is given to be:[5]

$$H_c = (H_{1x} * H_{x2})^{\frac{1}{2}} \quad \text{(Eq. 7)}$$

After this value is found, the microphones can be switched back, and data can be collected in the normal medium interface. The transfer function collected is divided by the calibration factor, giving the desired transfer function. This relationship is shown below:[5]

$$H_{12} = \frac{\widehat{H}_{12}}{H_c} \quad \text{(Eq. 8)}$$

The term $\widehat{H}_{12}$ is the uncorrected transfer function. Once this correction is made, the complex reflection coefficient is calculated using the following equation:

$$R(f) = \frac{e^{-ikx_2} - H_{12}e^{-ikx_1}}{H_{12}e^{ikx_1} - e^{ikx_2}} \quad \text{(Eq. 9)}$$

The term k is the wavenumber, and the x-values are the positions of the two microphones. With the complex reflection coefficient calculated, the speed of sound in terms of acoustic impedance can be deduced from the simple equation given below:

$$\frac{Z_m}{Z_w} = \left|\frac{1+R}{1-R}\right| \quad \text{(Eq. 10)}$$

The absolute magnitude of the quotient in this equation is an addition to the conventional impedance equation that was added so that only the real component of the sound speed is graphed. An imaginary component is present because the speed of sound is changing due to a frequency-dependent absorption level. By only analyzing the real component, the correct phase speed graph can be obtained. A good example of this graph that shows the theoretical curve is shown below:

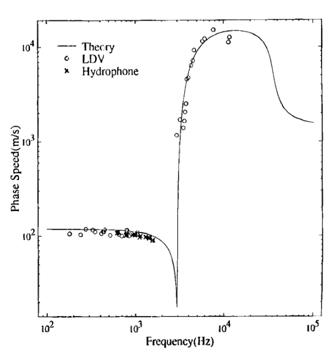

FIG. 4. Phase speed versus frequency (void fraction=1%, bubble radius =1.11 mm).

This graph was generated by Dr. Cheyne and his colleagues in their previous experiment while using a fiber optic measurement system.[3] The dip in the curve resembles the single bubble resonance. The sound speed through the bubble at this point is one to one to three orders of magnitude slower than lower or higher frequencies, respectively.

Materials and Methods

During this experiment, the sound being sent through the impedance tube was analyzed with a Stanford Research SR785 signal analyzer. This allowed us to obtain high resolution data sets, up to one hundred times more accurate than the oscilloscopes that would otherwise be used. Below is a diagram depicting the general setup of the system:

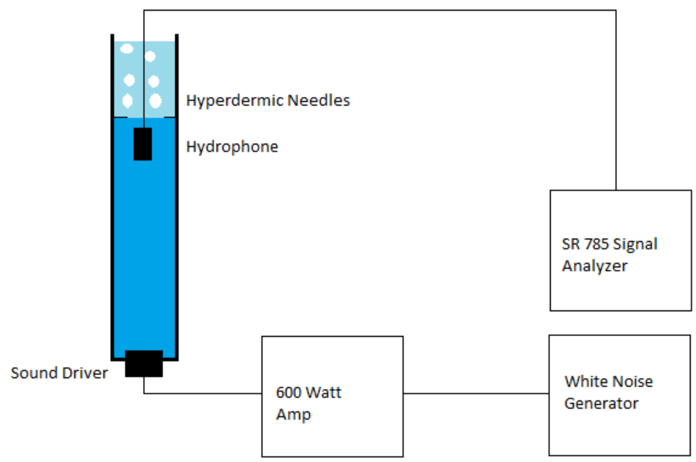

Figure 1: This is the general setup of the two single-hydrophone transfer function systems. Details such as the size of impedance tube and number of needles vary with each tube generation.

In this system, broadband sound is generated from the signal analyzer as white noise. The amplifier is used because the signal analyzer does not draw enough current to produce high sound pressure through the tube with the driver. Hypodermic needles are imbedded in the tube's wall and attached to a compressed air tank. By regulating the flow into these needles, we can regulate the average size of the bubbles travelling through the upper section of the tube. The sound inside the tube is recorded with a hydrophone and sent back to the signal analyzer. In order to computationally analyze the data being collected in the analyzer, a LabVIEW block program was created. This program allowed data to be queried out of the analyzer and collected in the computer as a two column data set, row number dependent on the designated number of 'FFT lines' being applied in the transfer function calculation, or overall resolution of the data collected. This data was exported and placed in an Excel spreadsheet that calculated the sound speed at every frequency in the broadband spectrum, and also generated the graph that was generated from the sound speed data.

In order to predict where the resonance phase speed is going to occur, the bubble size must be closely controlled. A flow meter is attached to the compressed air tank to regulate how much air is being put through the needles at any given time. The flow meter is essential to controlling bubble size because the bubbles are not visible inside the tube. So, in order to accurately measure size, hypodermic needles attached to this flow meter were placed inside a fish tank. With a strobe light, we could sync up the frequency of the light to the rate at which the bubbles entered the tank. If this is done properly, the bubbles appear to be stationary, resembling a time lapse image of a ball falling through the air. A ruler was placed directly beside the bubble field for scaling purposes, and with a CCD camera, this image was captured and stored onto a computer. The rate of flow was calibrated to the size of the bubbles by using this technique.

Our first generation tube in this experiment is a large, rigid, steel tube. The reason for this design is to have a tube that is very dense compared to water, and that also has enough area inside the tube for accurate sound pressure mapping. Because the tube is so large, the frequency limits exceed the 2 Hz-20 kHz range that is white noise. This allows us to map out the sound absorbing behavior of bubbles over a large frequency range. A single hydrophone used to collect the sound in the system is placed in the center of the tube, one half wavelength below the bubbly water medium. The sound in the tube is generated from a speaker-driven piston at the base of the tube. To generate planar waves in the tube, this piston drives a water-tight membrane that seals the tube's base. Shown at the top of the next page in the first chart is a chart with some design specifications of the tube.

Our second generation tube is a small scale version of the larger steel tube. This tube is a less rigid aluminum tube. We chose the aluminum tube because it was much smaller and we could easily move it around the lab as necessary. The smaller tube diameter is favorable because it allows the sound to form planar waves much easier than in the larger tube. Bubble size is more easily controlled, because although we only have two needles present in the tube wall, the total volume of the bubble medium section is much less than that of the steel tube. We can maintain the same void fraction with less flow in the needles, and lower flow correlates to more controlled bubble size. With a smaller tube diameter, our sound collection range dips to 1 kHz-12 kHz; however, smaller frequency ranges must be observed to record high resolution measurements, so this is not an issue. One flaw in this system is the hydrophone we are using. It is not very large in comparison to the steel tube, but it is a third the size of the inner diameter of this tube, making it a significant obstruction the path of the waves travelling through the tube. Shown at the top of the next page in the second chart is a chart of the specifications of this tube.

Our third generation tube was created to test out the dual-microphone transfer function method. Below is a conceptual image of the new system:

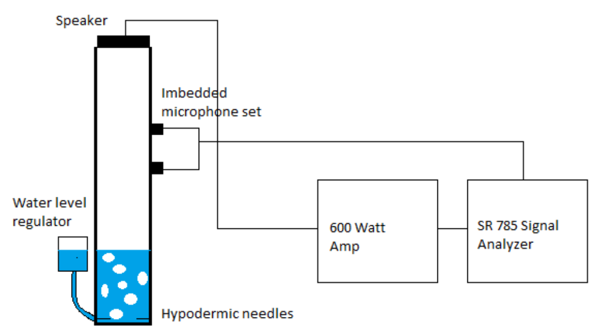

Figure 2: This is the conceptual image of the Plexiglas tube setup. It sends sound through an air to water-air interface. This time, two imbedded microphones inside the tube wall are used in place of the hydrophone.

An air to water-air interface was used in this setup because of the high impedance mismatch between air and Plexiglas. Although not intuitive, Plexiglas and air together have a higher mismatch than steel and water, as used in the hydrophone method. The impedance ratio between steel and water is 40:1, whereas the ratio between Plexiglas and air is 100:1. Microphones are imbedded into the impedance tube's wall that act together to collect the signal that is displayed in the analyzer. With one microphone being used, data collection this system is very efficient. With two, the data collection is not as simple. The phase mismatch between two microphones has to be negated with a microphone switching calibration technique, and is time consuming; however, it allows for quick data collection after calibration is complete. This system is not recommended for proof of concept measurements, but very useful if looking to generate a good data set in one specific tube location. A chart that shows the specifications of this impedance tube is shown in the third chart on the top of the next page.

Steel Tube						
Total length (cm)	Specimen length (cm)	Inner Diameter (cm)	Outer Diameter (cm)	Needle Count	Collection Method	Interface
230	25	5.5	7	12	single hydrophone	water--water/air

Aluminum Tube						
Total length (cm)	Specimen length (cm)	Inner Diameter (cm)	Outer Diameter (cm)	Needle Count	Collection Method	Interface
52	varied	3	3.8	2	single hydrophone	water--water/air

Plexiglas Tube						
Total length (cm)	Specimen length (cm)	Inner Diameter (cm)	Outer Diameter (cm)	Needle Count	Collection Method	Interface
60	varied	2.6	3	2	dual microphones	air--water/air

The concept of the one hydrophone math is simple: mathematically describe the geometric properties of the tube and cancel them out of the system. Some effects of the tube that need to be negligible include tube attenuation and resonating frequencies, especially ones that are dependent on water level inside the tube. Both of these effects can make the sound signal very difficult to read. To do this, the transfer function and acoustic impedance of the tube need to be measured at three arbitrary termination lengths, or water levels inside the tube. When measuring these values, a singular water medium interface is used. Equations 2-4 are all three equations of constants that are in terms of these six calculated values. The f_i values are all the transfer function values found by using the signal analyzer's function calculation tools. The y_i values are the acoustic impedance values calculated with each corresponding termination length's transfer function value.[5] After the three constants are calculated, the bubbles are put back into the system to create a two medium interface. The hydrophone is placed in any location that is one quarter wavelength below the bubble field, and the transfer function of the sound in the tube is taken at that position. This value is the y_o term in equation 1. With this value, the acoustic impedance of the system can be calculated, and thus, the phase speed through the bubble field is deduced.

The two microphone method is much different, however no more useful than the hydrophone method. Because two microphones are used, the geometric properties of the tube can be ignored much easier. By cross correlating the signals obtained from each microphone, the similarities of the signals are essentially divided out, leaving only the details that are unique to each microphone. These details that are observed are the changing sound effects that are a result of the bubble field interacting with the white noise traveling through it. The seemingly ubiquitous and constant background noise, contributed by tube attenuation and resonance frequency, is cancelled out of each microphone's signal. This method requires that each microphone is calibrated to the other because of a phase mismatch between them. To do this, the transfer function must be taken twice, both of the white noise traveling through a singular sound medium. First, the transfer function must be calculated with microphone one and two in positions one and two, respectively. These two positions are arbitrary within the region between the source and the bubble field. Second, the two microphones must be switched, and the process repeated. This calibration process is done to satisfy equation 5. The term H_{1x} is the transfer function of the microphones in the correct position, and the term H_{x2} is the transfer function of the switched position. This equation is used to calculate the correction factor of the system. This value is used to correct the transfer function value obtained from the two medium interface system that is being observed by dividing it into the new transfer function, as shown in equation 6. This value can then be used to calculate the acoustic impedance, which can then be used to find the phase speed through the bubbly liquid.

Data

Shown below is a collection of graphs generated from data collected in the SR785 analyzer, each displaying the void fraction and bubble size for the result shown:

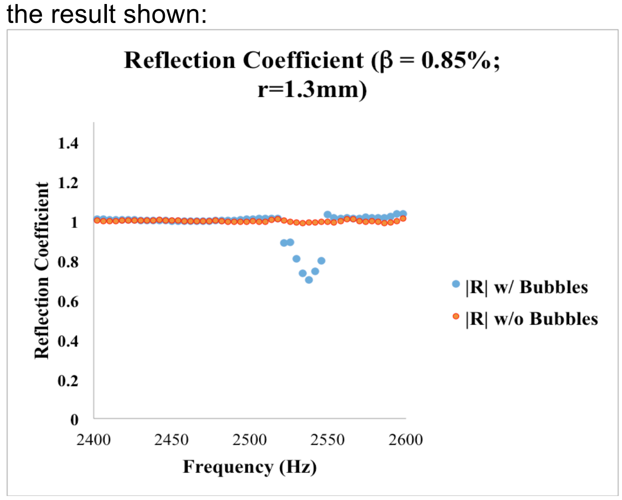

Figure 3: This is the reflection graph generated by a void fraction of 0.85% and a bubble radius of 1.3mm, making the resonance dip occur at 2.5 kHz. Notice the absorption dip in the curve with bubbles. This is the effect that one hopes to see in this experiment.

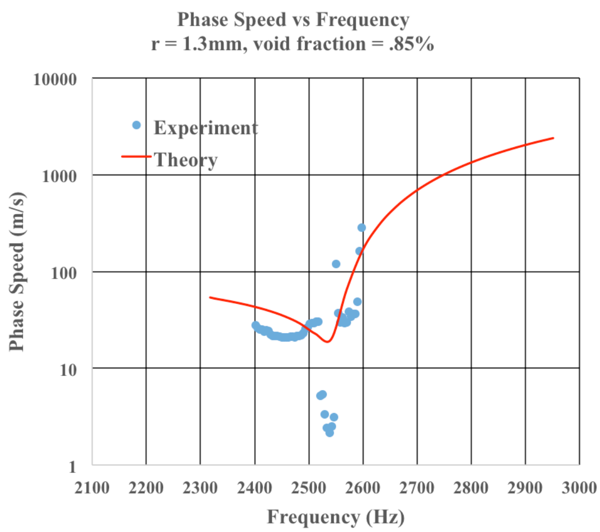

Figure 4: Here is the phase speed curve generated from the 0.85% void fraction run in Figure 3. The lowest phase speed value recorded here is at 2.133 m/s at bubble resonance.

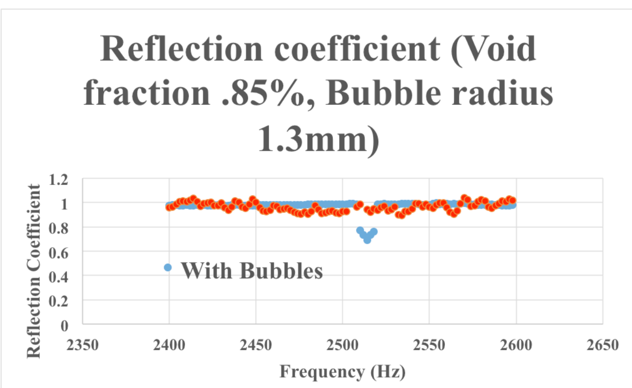

Figure 5: This is the reflection curve generated from another 0.85% void fraction and 1.3mm radius collection. Again, notice the dip in the 'with bubbles' graph.

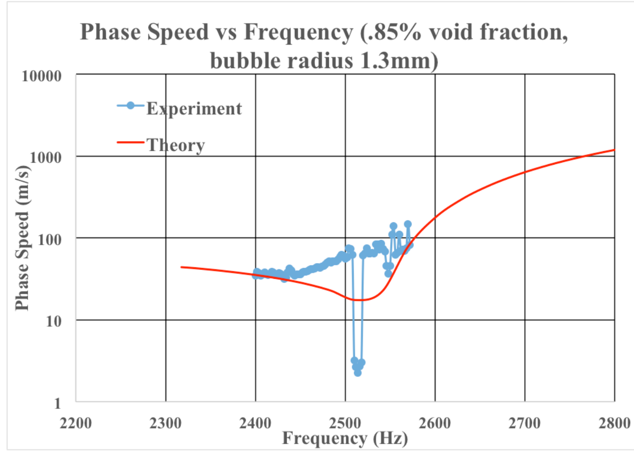

Figure 6: This is the phase speed curve generated from second 0.85% void fraction data collection shown in Figure 5. The lowest phase speed measured in this collection is 2.27 m/s.

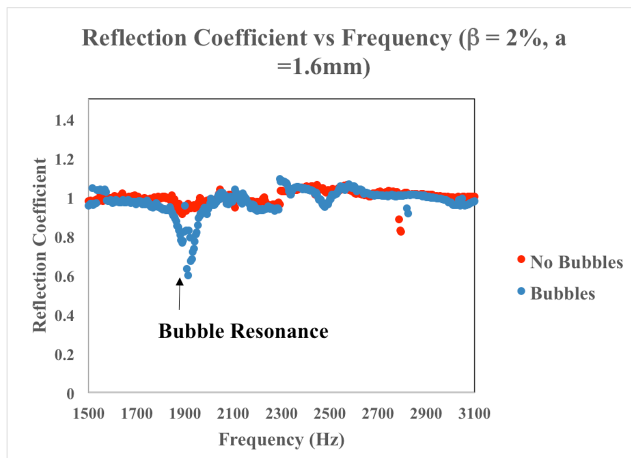

Figure 7: This is a reflection curve generated from a data collection with a void fraction of 2% and a bubble radius of 1.6mm. This bubble radius gives a resonance dip at 2 kHz.

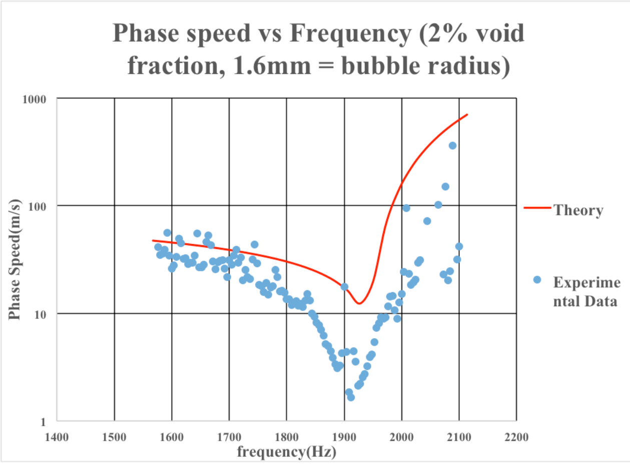

Figure 8: This is the phase speed graph generated from a 2% void fraction data run with a bubble radius of 1.6mm. The lowest phase speed measurement recorded is 1.66 m/s at 1912 Hz. Notice how the higher void fraction produces a clearer result.

DISCUSSION

In this experiment, acquiring the transfer function is necessary for calculating the phase speed of the bubbly liquid. Because it is a complex value, the transfer function can be expressed in two different values: phase and magnitude. The SR785 User Math function menu is used to calculate the transfer function inside the analyzer before the data is exported. The following function input is what is used to do this calculation:

$$SSUserF1 = \left(\frac{mag(spec(2))}{mag(spec(1))}\right) * \left(\frac{cross\ spec()}{conj\ spec()}\right)$$

This equation satisfies equation 2. This equation is applied to both channels in the analyzer, and the analyzer records the data in and FFT format. This allows the machine to take the incoming two signals, microphone 1 to channel one and microphone 2 to channel 2, and cross correlates them to give the transfer function in terms of its magnitude and phase.

With labview controlling the analyzer via GPIB connection, these to values are exported into a two column data file and stored on a computer. This data file can then be used to calculate the transfer function value for each frequency data point collected in Microsoft Excel. Below is an example of this calculation in the Excel equation syntax:

$$H_{12} = improduct\left(B\#, imexp\left(imcomplex(0, A\#)\right)\right)$$

In this equation, column A and B represent the imported phase and magnitude values, respectively.

The one microphone technique yielded a clear, conclusive result at two separate bubble sizes and void fractions. A void fraction of 2% and 0.85% was used for this method. With a bubble radius of 1.3mm and 1.6mm respectively, theory suggests that a resonance frequency between 1.8-2.5 khz was to be expected. In order to achieve a high resolution data set, a small frequency window around the predicted resonance frequency was observed. Looking at a small frequency window from 1.5-2600 Hz with 132 FFT lines, a resolution of 4 Hz between each data point was achieved. This method yielded a phase speed of 1.6-2.3 m/s at each respective bubble resonance value.

CONCLUSION

In this experiment, we measured the phase speed of a bubble field at the single bubble resonance. To do this, we used two separate transfer function methods and separate impedance tube setups. The results shown in this paper are those of the one microphone technique used on a Plexiglas tube with an air-bubbly water interface. The single bubble resonance frequency for a bubble field with 1.6mm radius bubbles was found to be 1800 Hz, which is 10% off the theoretical value of 2000 Hz. We also measured the single bubble resonance frequency for a bubble field consisting of 1.3mm bubbles at 2.55 kHz, which is 4% off the predicted resonance frequency. Unfortunately, the supersonic speeds were not achieved at high frequencies for either bubble curve, partially due to the higher void fraction being used. This is not a big loss, however, because the high and low frequency domains have been measured many times by many researchers. Despite the present errors, we were able to measure a resonance phase speed that is one fifth the theoretical speed given by the known model. Some possible explanations for this error in the phase speed can be found by looking at two things. First, the impedance mismatch between water and air may be too great to be negligible. Second, the attractive forces between the bubbles at high void fraction may be too strong to consider this a linear situation, as the theory suggests.

Future Work

This experiment showed us that we are able to actually observe the behavior of sound as it travels through a bubble field. As experimentalists, we aim to be able to provide data sets, experimental parameters, and system specifications to theorists so that they can build predictive models for this field of study. This semester, I have spent time continuing my research in this field and helping Professor Cheyne and Professor Thurman finish this project. Our short term goal has almost been reached—replicate this result close to theory and show that measuring the phase speed at resonance is possible. Our long term completion goal is to be able to provide a researcher with a step-by-step guideline to measuring the correct phase speed graph, and a detailed description of each method that allows one to know which system best applies to any given situation. Being able to measure the phase speed in high resolution as we did this semester, we strongly believe that this goal can be reached.

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