# Observing the Dispersion Relation of a Complex System of Gravitational and Capillary Waves

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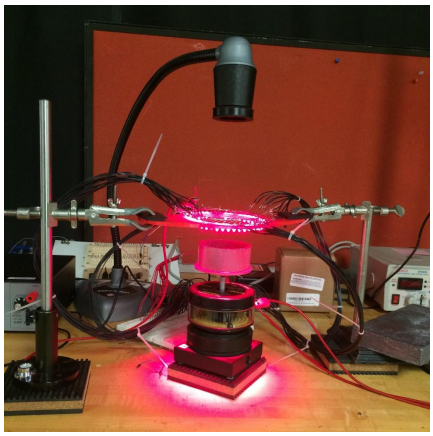
## ABSTRACT

The objective of this experiment was to measure the wavelength of water from the resultant pattern of capillary/gravitational waves driven by an oscillating speaker. These results would then be used to compare and validate the results of Carson Maki’s experiment. Then isopropyl alcohol was measured in order to validate this form of measurement for other substances. Both materials produced a relationship between the wavelength and the frequency that matches that of Maki’s.

## INSTRUMENTATION AND EQUIPMENT

The experimental setup required a deep cylindrical dish that was attached to a speaker. This speaker was driven vertically by a function generator. A digital camera was positioned directly above in order to image the wave patterns. A circular array of red LED lights was then placed above the dish to assist in the imaging process. Under the speaker and LED array and table, several blocks of Rubber/cork anti-vibration pads were placed. These were used to reduce vibrations effect on waves and imaging. An example of this setup can be seen below:



## THEORY

The dispersion relation of waves on liquid surfaces can be written as

$$w^2 = (gk + \frac{\gamma k^3}{\varsigma}) \qquad (1)$$

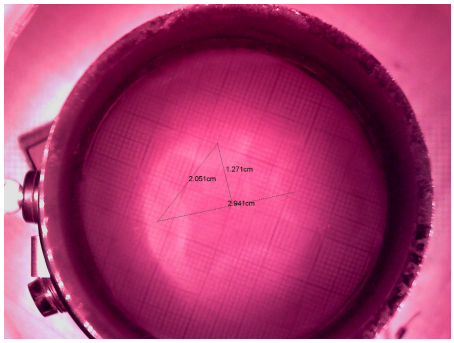
Based on Carson Maki’s work, if the gravity waves are completely absent from the system, then the system can be expressed as

$$Log\omega = \frac{-2}{3} Log\omega + Log\frac{\gamma}{\varsigma} \quad (2)$$

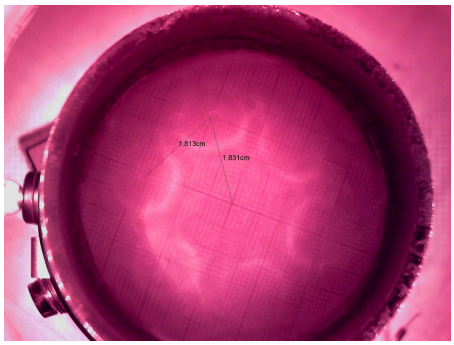
On the other hand, the slope is -2 when it is solely a gravitational system. Since we were observing a complex system influenced by both of these systems, we were looking for a slope where

$$-2 \leq m \leq \frac{-2}{3} .$$

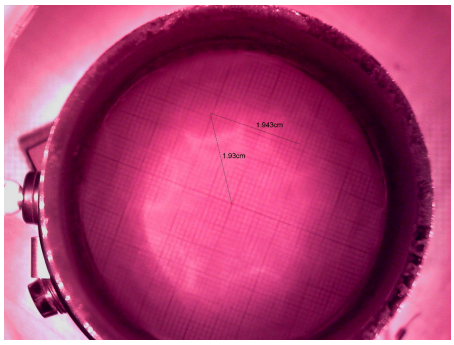
## DATA



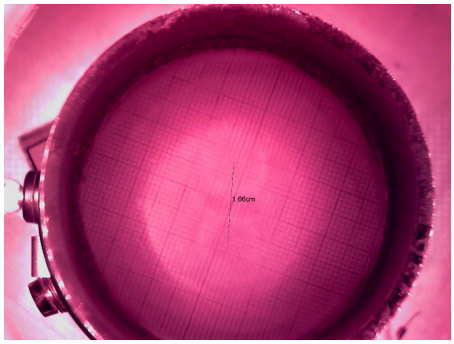

13.9 Hz



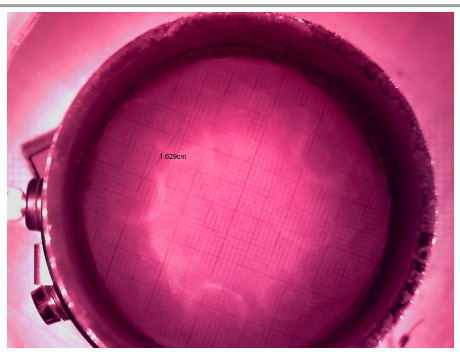

15.8 Hz



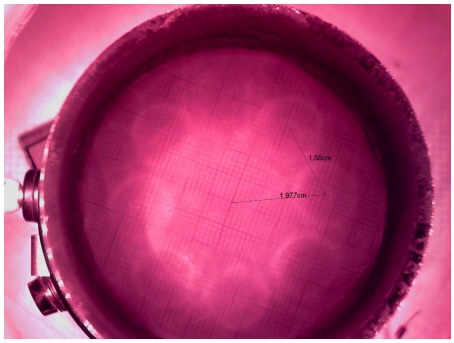

16.37 Hz



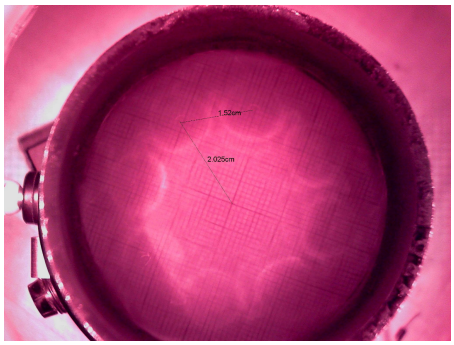

16.7 Hz



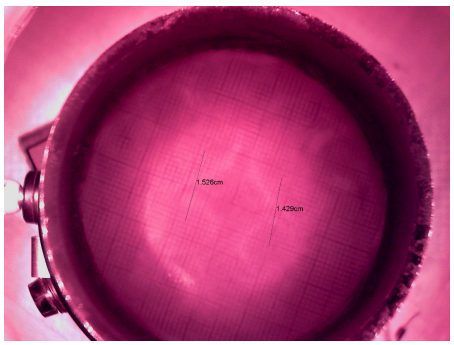

17.2 Hz



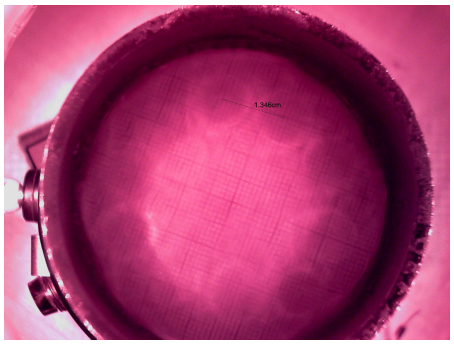

17.6 Hz




18.43 Hz

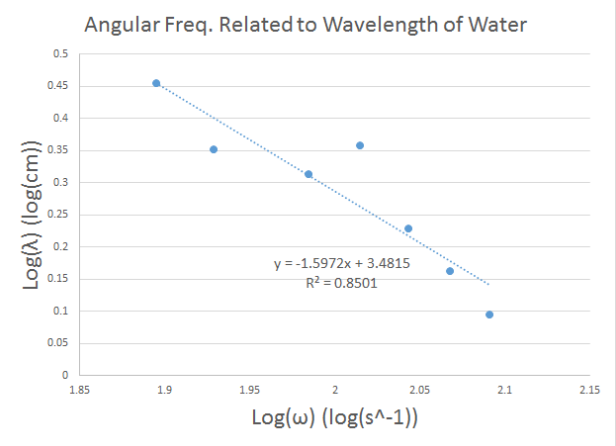



19.08 Hz



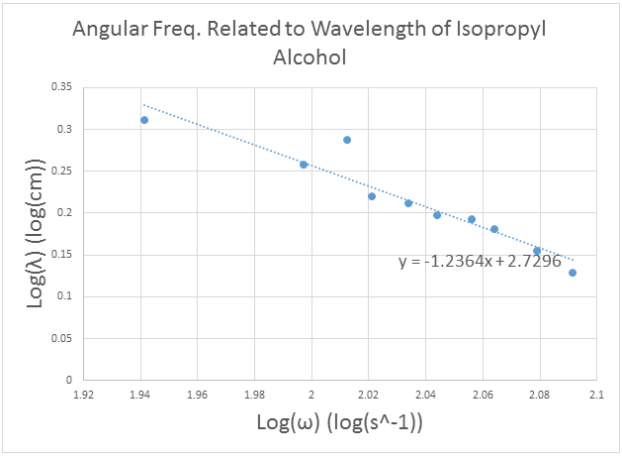

19.64 Hz

| Water | | | | |
|---|---|---|---|---|
| Frequency(Hz) | wavelength (cm) | W (freq*2*pi) | Log(W) | Log(wavelength) |
| 19.62 | 1.249 | 123.2759916 | 2.090878505 | 0.096562438 |
| 18.59 | 1.456 | 116.8043162 | 2.067458891 | 0.163161375 |
| 17.56 | 1.698 | 110.3326408 | 2.042704013 | 0.229937686 |
| 16.46 | 2.288 | 103.4211428 | 2.014609332 | 0.35945602 |
| 15.36 | 2.061 | 96.5096448 | 1.984570717 | 0.314077992 |
| 14.45 | 1.258 | 90.791951 | | |
| 13.5 | 2.251 | 84.82293 | 1.92851327 | 0.352375495 |
| 12.5 | 2.856 | 78.53975 | 1.895089515 | 0.455758203 |
| 11.5 | 1.276 | 72.25657 | | |
| 10.5 | 1.372 | 65.97339 | | |
| 9.5 | 1.179 | 59.69021 | | |




| Isopropyl Alcohol | | | |
|---|---|---|---|
| Frequency (Hz) | wavelength (cm) | Second Measurement | K value( 2*pi/wavelength) |
| 11.5 | 0.23 | | |
| 12.2 | 0.53 | | |
| 12.6 | 0.818 | | |
| 13.2 | 0.968 | | |
| 13.9 | 1.27 | 2.051 | 306.3471477 |
| 15.8 | 1.831 | 1.813 | 346.5626034 |
| 16.37 | 1.93 | 1.943 | 323.375193 |
| 16.7 | | 1.66 | 378.5048193 |
| 17.2 | | 1.629 | 385.7077962 |
| 17.6 | 1.977 | 1.58 | 397.6696203 |
| 18.1 | | 1.563 | 401.9948816 |
| 18.43 | 2.02 | 1.52 | 413.3671053 |
| 19.08 | | 1.429 | 439.6906928 |
| 19.64 | 1.346 | 1.346 | 466.8038633 |

| | | | | | With new measurements | |
|---|---|---|---|---|---|---|
| W (freq*2*pi) | | Log(W) | Log(wavelength) | | Log(W) | Log(wavelength) |
| 72.25657 | | 1.858877342 | -0.638272164 | | | |
| 76.654796 | | 1.884539332 | -0.27572413 | | | |
| 79.168068 | | 1.898550047 | -0.087246696 | | | |
| 82.937976 | | 1.918753433 | -0.014124643 | | | |
| 87.336202 | | 1.941194302 | 0.103803721 | | 1.941194 | 0.31196566 |
| 99.274244 | | 1.996836588 | 0.262688344 | | 1.996837 | 0.258397804 |
| 102.8556566 | | 2.012228181 | 0.285557309 | | 2.012228 | 0.288472801 |
| 104.929106 | | 2.020895973 | | | 2.020896 | 0.220108088 |
| 108.070696 | | 2.033707948 | | | 2.033708 | 0.211921084 |
| 110.583968 | | 2.043692169 | 0.296006669 | | 2.043692 | 0.198657087 |
| 113.725558 | | 2.055858076 | | | 2.055858 | 0.193958978 |
| 115.7990074 | | 2.063704837 | 0.305351369 | | 2.063705 | 0.181843588 |
| 119.8830744 | | 2.078757872 | | | 2.078758 | 0.155032229 |
| 123.4016552 | | 2.091320985 | 0.12904506 | | 2.091321 | 0.12904506 |




## ANALYSIS

Originally, the LED was not used in the experimental setup. Imaging, however, proved to be too difficult as the camera was unable to differentiate the peaks of the waves. An LED array from a previous experiment was found, however, the circuit used for the array had to be remade. A circuit of many 180 Ω resistors in parallel was created to regulate the light setup. Images were then able to be taken and measured using the Accuvision software of the camera. The Accuvision software calibrates to the proper scale and can then make accurate measurements off of the image itself. The images above show how measurements of the peaks of the variously patterned wave arrays were taken. This experiment was measuring the wavelength of patterned arrays versus the driving frequency in the system. First measurements were taken of water, as was done in Carson Maki's experiment. These results were then compared to the results Maki received on his experiment. Afterwards, the experiment was conducted again, this time with isopropyl alcohol. The change in material was done to further test if the theoretical dispersion relation for a complex system can work on other liquids. Both of our results fell within the desired range. Water produced a slope of -1.59 while the isopropyl alcohol produced a slope of -1.23. Both of these results suggest that the patterns observed were driven by a complex system of capillary and gravitational but suggest that the gravitational waves are more influential in this system.

## CONCLUSION

From the data gathered, we were able to confirm the results of Carson Maki's work. Also, with the results of the isopropyl alcohol, it has been confirmed that this method of measuring capillary waves is effective with other substances. For future work, there is opportunity to image these waves with a strobe and a stronger function generator. It is also possible to move on from here and observe Faraday waves or multi layered liquids in the same manner.

## REFERENCES

Carson Maki. Faraday Waves on a Periodic Substrate. Hampden-Sydney College Physics Department 1-33, 2015.

Klemens PG. Dispersion relations for waves on liquid surfaces. American Journal of Physics 52: 451–452, 1984.