The Future of Flight: Using MATLAB Code to Determine Feasibility of Electric Flight

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Abstract

The purpose of this research was to determine the feasibility of electric and hybrid aircraft. This was done by taking available data on current battery and aircraft design and creating a MATLAB code in which predictions can be made for wing shape based on parameters necessary for the aircraft. Equations governing airfoil design were coded into MATLAB in order to provide a collection of different airfoils that could be tested based on the restraints provided for electric flight. These equations were specific to the NACA four-digit airfoil series which allowed for 10,000 different airfoil shapes to be tested. Once this was done, the coefficient of lift for each NACA number then needed to be calculated. This was done using the Hess-Smith panel method. Once these values could be determined, the parameters of the aircraft were prompted for, allowing the code to determine the necessary coefficient of lift for the aircraft. Once this coefficient of lift was found, the MATLAB code sifted through the thousands of different airfoil designs in order to find ones with similar coefficients of lift and output these designs. This code can now be used to theoretically determine the two-dimensional NACA airfoil that would work best for a certain electric aircraft.

Introduction

Air travel is one of the most common ways for people to travel vast distances in the modern world; it is fast and convenient which is why it is a billion dollar a year industry and is crucial to the global economy. However, air travel creates carbon emissions which are increasingly harmful to the environment. In a New York Times article from 2013 author Elisabeth Rosenthal estimated that for each flight from LA to NY or NY to Europe the average carbon emission “has a warming effect equivalent to 2 or 3 tons of carbon dioxide per person” [2]. This shows how impactful one flight can be to the Earth’s atmosphere and to the progression of climate change. Because flight is so common and frequent, these emissions continue to get worse daily and will only worsen climate change. Along with this fact, in the near future humanity will run out of fossil fuels, as we are using them faster than they can be produced, and industries must adapt to this change.

The solution to these inevitable realities is one that has already made waves in the auto industry and is soon to make waves in the airline industry, hybrid and fully electric machines. With the advent of hybrid and electric automobiles coming to the market the airline industry is not far behind. The science required for this advent is on the brink of overturning the industry and research into its abilities and capabilities are crucial to its fruition.

Theory

The theory behind this experimentation primarily comes from the determination of lift values over two dimensional airfoils. Because two-dimensional airfoils are infinitely thin, the most common practice in determining their lift force is by using the equation

$$L = 2\pi\alpha \qquad (1)$$

Where alpha is the angle of attack. This is useful for most approximations when it comes to two-dimensional aircraft but is not fully accurate in determining specific lift values for specific airfoils. There are a multitude of ways in which the most accurate lift values can be determined for different airfoil shapes, these include but are not limited to conformal mapping, and a variety of what are called panel methods. In these panel methods the airfoil itself is broken into panels allow important forces at each panel to be calculated and summed up, giving the lift force. The panel method used in this experimentation was the Hess-Smith panel method, which through linear algebra and summation of strengths, the lift force was determined. This method calls for a variety of in-depth calculations of vortex and source strength at each panel. In an attempt to get the most accurate calculations and not reinvent the wheel, a previously written code for lift values using the Hess-Smith panel method was used and altered to fit the airfoils input into the code. In order to get the shape of the airfoils, the NACA equations that govern the shape of NACA airfoils representing the corresponding NACA numbers was necessary. These equations are the following:

(2) $y_t = t\left(1.4845\left(\frac{x}{c}\right)^{.5}\right) - \left(.63\left(\frac{x}{c}\right)\right) - \left(1.758\left(\frac{x}{c}\right)^2\right) + \left(1.4215\left(\frac{x}{c}\right)^3\right) - \left(.5075\left(\frac{x}{c}\right)^4\right)$

(3) for $0 < x < x_a$
$y_c = (y_a/(x_a{}^2)) * (x) * (2x_a - x)$

(4). for $x_a < x < c$
$y_c = \left(\frac{y_a}{(c-x_a)^2}\right) * (c-x) * (c + x - (2 - x_a))$

(5) $y = y_c + y_t$

Where y is the thickness of the airfoil at a given position along the cord, x, from zero to where the cord ends, c. t is the thickness of the airfoil which is determined using the last two digits of the NACA number. x_a, is the value on the x axis where the max y value y_a occurs. y_t is the thickness along the mean camber line, while y_c produces the mean camber line. Adding the thickness to the mean camber line both above and below the mean camber line allows for the two dimensional airfoil to be created. Each NACA number has an important aspect of these equations within it. From the first digit, the y_a value can be derived, from the second digit, the x_a value can be derived. Finally from the last two digits of the NACA number the thickness can be derived.

Once these equations were coded into the MATLAB program 2-d airfoils were able to be printed by simply inputting the desired NACA number. These output airfoils were as precise as the same airfoil produced through other techniques. This then allowed for an accurate compilation of every single NACA 4 digit airfoil to be produced. From this, the code for determining the lift over a two dimensional airfoil was needed. This code included the use of the Hess-Smith panel method in which the source and vortex strength over each panel is calculated, and summed up. Once this is done a matrix of the varying source and vortex strengths at each panel is created allowing for the coefficients of lift and drag to be calculated. For a much more in depth explanation of the theory and mathematics behind this technique see reference [6]. Once this was done and the Coefficient of Lift was calculated it could be compared to the Coefficient of Lift necessary for a certain electric aircraft to fly. This Coefficient of lift was calculated using the following equation:

(6) $C_l = L/.5\rho v^2$

Where L is the lift force, ρ is the density of the air and v is the velocity of the airflow over the airfoil. The velocity was calculated from the following equation:

(7). $v = (P * 0.9)/T$

Where v is the velocity, P is the power that the motor can produce in KW, and T is the thrust that the motor propeller system can produce. Because the lift force is equivalent to the weight of the aircraft the lift force can be calculated by simply figuring out the mass of the entire aircraft including motors and batteries, once this is done and the air velocity that the propeller motor system can produce is found, the necessary Coefficient of Lift can be found.

Experimentation

The first step in the experimentation aspect of this research was to make sure the code for the four digit NACA numbers worked and was accurate. This was done by taking the output airfoil created and comparing its shape to the same airfoil generated by a known working code. In Figure 1 the two airfoils can be seen side by side.

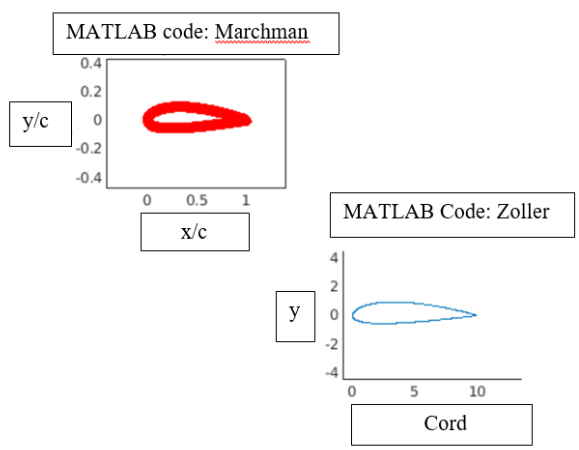

Figure 1: *Known code vs experimental code for NACA airfoil*

Marchman's code is an output from a Virginia Tech professor Dr. Marchman whose airfoil shape was used as an error check on the experimental code.

Once the airfoil generating code was checked for accuracy the Hess-Smith code for finding the coefficient of lift for a two-dimensional airfoil needed to be adapted to fit the experimental airfoil code. This was done by probing through Dr. Marchman's code and making changes where necessary in order to make the two codes cohesive. In order for the two codes to work the experimental code had to be altered slightly so that the airfoil shape was being created with one distinct line rather than multiple. This allowed for the airfoil to be broken up into panels along the entire shape. Once this was done the coefficient of lift for all NACA four digit airfoils could be calculated.

After calculations were able to be made available for all 10,000 four digit NACA airfoils the code had to be further altered to sift through every single one and determine the Coefficient of Lift for each at given angles of attack. Once this was done further code was written that asked the user the type of aircraft he/she wanted to find airfoils for. These questions included the following: What is the mass of the motor? How many motors are being used? What is the mass of your propeller? How many propellers are you going to use? How much power can each motor produce in KW? How many kgs of thrust can each prop motor system produce? What is the necessary battery mass needed to run these motors? What is the mass of the cargo? Asking these questions allows the user to input the specifications of his/her specific electric aircraft.

Once the values that are associated with these questions are received they are sent to equations 7 and 6 in order to determine the necessary coefficient of lift for the specific aircraft. From here the 4 digit NACA code begins cycling through all the 4 digit NACA airfoils in order to determine which, if any, 4 digit NACA airfoils have a coefficient of lift within plus or minus .005. If this is the case the code then outputs the 4 digit NACA number.

Data

The output of the final MATLAB code was dependent on the accuracy of the coefficient of lift values produced by the combination of the Hess-Smith and experimental NACA 4 digit code. In order to determine the accuracy of this code, airfoils with known coefficients of lift values that were

Angle of Attack	NACA #	Wind Tunnel Data Cl	Code Cl	% discrepancy
10	"0006"	0.8	0.889	11.125
12	"0006"	0.8	1.2385	54.8125
10	"0009"	1	0.9466	5.34
12	"0009"	1.295	1.2892	0.447876448
10	"1408"	1.1	0.881	19.90909091
12	"1408"	1.3	1.325	1.923076923
10	"1410"	1.1	0.9247	15.93636364
12	"1410"	1.375	1.3454	2.152727273
10	"1412"	1.1	0.9667	12.11818182
12	"1412"	1.38	1.3578	1.608695652
10	"2412"	1.23	0.92451	24.83658537
12	"2412"	1.4	1.4016	0.114285714
10	"2415"	1.2	0.9826	18.11666667
12	"2415"	1.4	1.4104	0.742857143
10	"2418"	1.15	1.0334	10.13913043
12	"2418"	1.4	1.4168	1.2
			Avg % discrepancy	11.28268987

Table 1: *Experimental wind tunnel Coefficient of Lift values vs. Experimental Code output Coefficient of Lift Values.*

experimentally tested in wind tunnels were compared to the output the experimental code produced. Table 1 shows the comparison between these values.
This data shows that the code is accurate for angles of attack of ten and twelve degrees with a mean percent discrepancy of 11.2%. This provided a solid base with which the final code could work off of.

The data from the final code output NACA numbers with coefficients of lift that were within plus or minus .005 that of the necessary values required to lift the type of electric aircraft the user input. An example of these airfoils being output can be seen in Figure 2.

```
>> Finalelectricaircraft
What is the mass of the motor?
.15
What is the mass of the motor?
2
What is the mass of your propeller?
.05
How many propellers are you going to use?
2
How much power can each motor produce in KW?
.636
How many kgs of thrust can each prop motor system produce?
1.17
What is the necessary battery mass needed to run these motors?
.333
What is the mass of the cargo ?
.5
How many motors are being used?

2003

2004

2005

8101
```

Figure 2*: Input and output of final code.*

In figure 2 values for a small scale RC aircraft were used in the hopes of building one and empirically testing whether the wings produced by the code will actually lift the aircraft off the ground. This output shows that any given design of an electric aircraft with propellers can be tested to determine whether or not it is feasible for the most common types of wing designs.

Conclusion

The data shows that the final code is accurate in not only determining whether or not an electric aircraft design will work, but what the best wing designs are for those that are indeed feasible. This accuracy is backed up by the fact that the underlying code producing these wing designs agrees very closely with the real world experimental data, with only a percent discrepancy of approximately 11%. Due to these assurances in accuracy this code can be further used to help determine the most efficient electric aircraft possible with the absolute best coefficient of lift needed for takeoff scenarios. Further research must be done in order to confirm this accuracy such as building an electric aircraft based on the values the code spits out and testing this aircraft experimentally for feasibility and productivity.

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