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## Realism in Messiaen's *Oiseaux Exotiques*: A Correlation

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In addition to being a great composer, Olivier Messiaen was a well-known amateur ornithologist, with particular fascination with birdsong. Indeed, in several of his compositions he used birdsong as a model, either for specific passages, or for entire pieces. While musicologists have differed on the accuracy of his claim, there have been relatively few attempts to rigorously quantify the extent to which Messiaen's music is similar to the birdsongs that inspired it. The research project focuses on comparing Oliver Messiaen's compositions in the piece *Oiseaux Exotiques* to the actual birdsongs which were the inspiration for it. Mathematical tools of analysis such Fourier transforms and time-frequency-energy spectrograms were used to identify similarities between the birdsong and corresponding musical snippets. Matlab was the main tool for analysis used in this research project and a major aspect of the project was to code up the mathematical and signal processing theory and algorithms into the Matlab environment. One of the key ways by which correlations were accessed was through the use of "contours" which were extracted from the spectrograms. The results showed significant amounts of similarities and good correlation values between the specific musical snippets that were transcriptions of birdsongs. Statistical significance of the results was determined by running the correlation tests on 100 random snippets of music from *Oiseaux Exotiques*.

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# 1 Introduction

One of the most famous composers of the 20th century, Olivier Messiaen, drew some of his greatest inspirations from birds. Messiaen, a devout Catholic throughout his life, saw birds as perhaps the greatest musicians with abilities that seemed divinely endowed. He studied birds from all around the world and captured some of their magic in the form of musical transcriptions in his work which he claimed were “parfaitement authentiques” [2]. In fact, while most musicologists agree that some of Messiaen’s compositions do bear striking artistic similarities to real birdsongs, there has been little in the way of substantiating Messiaen’s claim with rigorous analysis to quantify such claims. The research presented in this paper looks to do just that. In essence, it focuses on testing several sections of Olivier Messiaen’s *Oiseaux Exotiques* by comparing specific birdsong transcriptions in it to actual birdsong recordings used to compose those pieces. This project aims to extend upon the research done on the same subject by a musicology professor and a Messiaen scholar at Cornell University, by employing a much more mathematical approach to quantitatively measure and evaluate the correlations that may arise between the birdsong and the music.

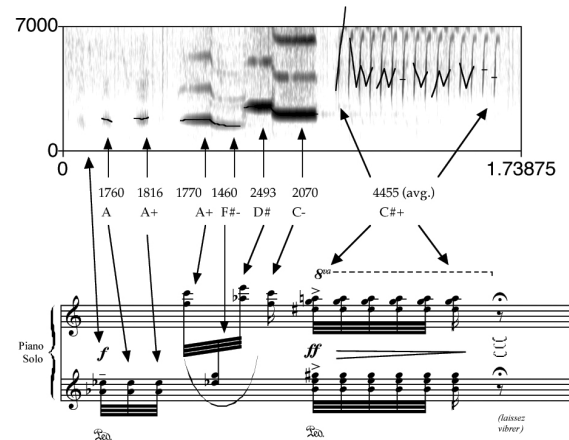
## 2 Methodology

The following sections give a general overview of the progression of this research and the specific kinds of analyses performed.

### 2.1 Fallon Pairings

The most important piece of preliminary research for this project is an article by a Dr. Robert Fallon. “The Record of Realism in Messiaen’s Bird Style”, that focuses on five American birds whose transcriptions are found in Messiaen’s *Oiseaux Exotiques*. It compares their birdsong to specific and corresponding pieces of the Messiaen’s music. Dr. Fallon used the actual 1942 birdsong recordings that Messiaen himself used when composing his piece on “Exotic Birds”. This research project mirrors Dr. Fallon’s work in significant ways by trying to compare Messiaen’s *Oiseaux Exotiques* music to birdsongs, but it is also different in a key way: mathematical and strictly objective means are adopted to determine the correlations (if any) between the music and birdsong. Figure 1 shows one of the figures from “The Record of Realism in Messiaen’s Bird Style” that helps to show the analysis done in that article.

Wood thrush, spectrogram and *Oiseaux exotiques*, p. 5

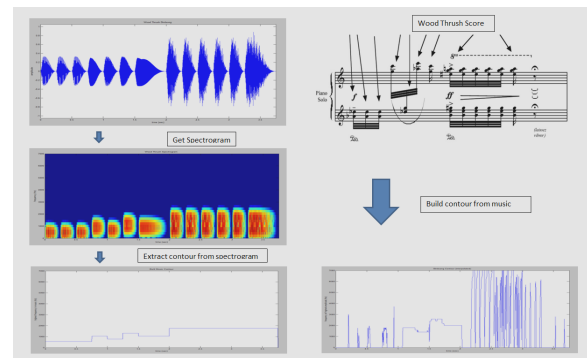


**Figure 1.** This figure highlights the similarities found between Messiaen’s music on the Wood Thrush and the actual birdsong. Dr. Fallon points out frequencies similarities between the spectrogram and the music score [2]. In contrast, this research paper focuses on mathematically computing and comparing the contours of the music and the birdsongs

Dr. Fallon’s work provided the pairings of the music and birdsong that serve as the focus of this research, and it also serves as a starting point for this research which aims to refine his analysis.

### 2.2 Contours and their Correlation

Dr. Fallon’s article provides seven pairings of the music and birdsong. The birdsongs are that of the Prairie Chicken, the Baltimore Oriole, the Lazuli Bunting, the Woodthrush, and the Cardinal (which had three distinct parts to its birdsong and thus three pairings). For each birdsong and its corresponding music section, a time-frequency energy spectrogram is analyzed. The spectrogram is useful in extracting a “contour” for the birdsong. The contour for the corresponding piece of music is built based on the conductor’s score sheet for *Oiseaux Exotiques*. Figure 2 helps to illustrate this progression.



**Figure 2.** The process of deriving contours from the birdsong and from the music. The contours simplified

the large frequency data, allowing it to be correlated with each other.

These contours are important as they help to simplify the time-frequency-energy spectrograms of the signals into one-dimensional vectors which could effectively be tested for possible correlations. The three different methods by which the contours of a pairing are compared are centered correlation, uncentered correlation and CSIM correlation. Uncentered correlation is a fairly common way of finding the angle between two vectors or determining how close they are to each other. This correlation rests on a very important theorem in mathematics, the Cauchy-Schwarz inequality, which will be proven in the course of this paper. Centered correlation centers the contours to a mean of zero before finding a correlation between them. The final method of correlating the two vectors, CSIM correlation, is a method that is used in Music Theory when comparing the contours of the two different pieces of music [1]. Thus, for each of the seven pairings of birdsong with music signals, there are three different values of correlation that are derived.

### 2.3 Control Pairings

The results that are achieved through the correlation methods would be meaningless if there is not something to compare or measure them up against. The method by which this research takes into account the statistical significance of the results is by having certain pairings which are determined to be the control. Due to restraints of time, it was not possible to compute the statistical significance for all of the seven pairings. It was computed only for the Baltimore Oriole. The control is twenty random sections of music from *Oiseaux Exotiques* paired with the Baltimore Oriole birdsong. Any section which contained transcriptions from the Baltimore Oriole birdsong was carefully avoided. For each of these twenty pairings, the three measures of correlation are calculated. In the end there are 21 pairings, (including the one experimental result), for the Baltimore Oriole. The three values of correlation for the experimental results are compared against those of the randomly chosen pairings to determine statistical significance, (for the Baltimore Oriole). It would have been ideal to calculate the statistical significance for each of the birdsong, but as mentioned above, time became a limiting factor.

### 2.4 Methods and Tools

The great portion of this research revolved around coding up algorithms, classes and functions in Matlab and Netbeans, and of course debugging. There was not a lot of programming done in Java as a lot of previous code from the BareBonesMusic package written by Professor Pendergrass was used. However there were

two new classes that were written in BareBonesMusic, ControlMeasures and MusicDemos, which dealt with accessing and returning music contours. The bulk of coding was done in Matlab and the Java packages were imported into Matlab via .jar files. Near the later half of this project, a lot of the long spaghetti code was simplified into many different functions that performed specific tasks. For example the three similarity measures had their own specific functions; there were functions written to extract the contours from the spectrograms of the birdsong and build the contours from the music signal; another important function written was a shifting function which stretched and shifted one contour with respect to the other to find the best correlation between the two. Matlab was also very used to represent the results in meaningful plots.

## 3 Mathematical Background

### 3.1 Time Frequency Analysis

A major mathematical topic that helped to lay the foundation for this project was Fourier analysis. It might be useful to review the concept of what Fourier analysis is and how it is used. Fourier analysis, named in recognition of the famous French mathematician Joseph Fourier who helped to develop this vastly important field of mathematics, deals with taking functions that are represented in the time domain and deriving a frequency-domain representation for the signals/functions. This process works in the reverse also; that is Fourier analysis can be used to extract the time-domain representation of a signal from the frequency-domain representation. Taking a function of time and representing it as a function of frequency is very useful, especially in the field of signal processing. Through Fourier analysis one can take virtually any kind of complex waveform and break it down into sums of simpler trigonometric functions. The mathematical transform that performs Fourier analysis is called the Fourier transform and Matlab implements a very efficient algorithm, FFT, which is able to compute the discrete Fourier transform of a vector quickly. Below are the formulas used to compute the Fourier transform of a signal and the Inverse Fourier transform, which can convert a function back to a time domain representation from a frequency domain representation.

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-2\pi ift} dt \quad (1)$$

$$s(t) = \int_{-\infty}^{\infty} S(F)e^{2\pi ift} df \quad (2)$$

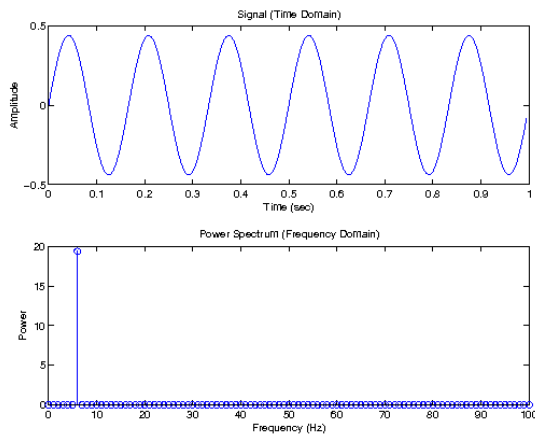
The frequency-domain representation of a signal is often a complex valued function. While the complex valued function is useful in many cases, often it is helpful

to just determine the energy of the various frequencies represented in the frequency-domain. The method by which this is achieved is by taking the energy spectral density (ESD) of the function. The ESD is a function of frequency that returns the energy present in a waveform due to specific frequencies. So, if the waveform is a simple sinusoid with a constant frequency, all of the energy of the waveform would be concentrated at that frequency. The formula for finding the ESD of a function is as follows.

$$ESD(f) = |S(f)|^2 = \left| \int_{-\infty}^{\infty} s(t)e^{-2\pi ift} dt \right|^2 \quad (3)$$

Where  $S(f)$  is a the frequency domain representation and  $s(t)$  is the time domain representation.

Figure 3 shows the ESD of a basic sinusoid function.



**Figure 3.** This figure shows a sine wave and its power spectrum. The Fourier transform of a function is often represented in this manner.

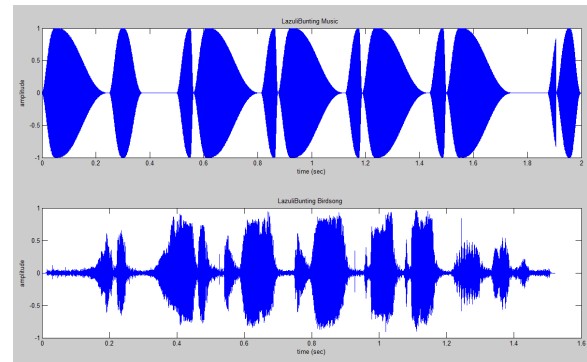
The formula for the energy spectral density is derived through Parseval’s theorem. This theorem hints toward the Fourier transform being unitary, which means that the square of the integral of a function is equal to the square of the integral of the transform of that function. Thus, the amount of energy in  $s(t)$  is equal to the energy in  $S(f)$ .

**Theorem 1** (Parseval’s Relation). *Let  $s(t)$  be the time domain representation of a function, and  $S(f)$  be its frequency domain representation,*

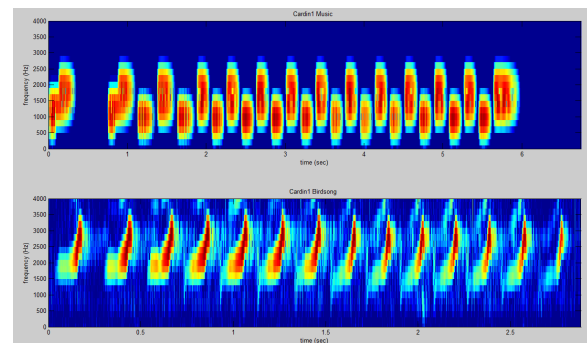
$$\int_{-\infty}^{\infty} s^2(t)dt = \int_{-\infty}^{\infty} |S|^2(f)df \quad (4)$$

The spectrogram is based on a clever use of Fourier transform that allows one to see the change in frequency intensity in the original time-domain signal as a function of time. In essence, it is calculated by computing the Fourier transform not on the entire time-domain signal but rather on small consecutive time

slices of that signal. This results in an image that has time on the horizontal axis and frequency on the vertical axis, and color intensity representing power by highlighting the frequencies that the Fourier transform showed to have high energy (for that specific time slice of the original signal). Spectrograms of the birdsong and the music can provide strong visual clues to the similarities between the two signals. In this research, correlations between the spectrograms were one of the factors that inspired attempts to quantify the relatedness of Messiaen’s music to actual birdsong. Figures 4 and 5 try to illustrate this.



**Figure 4.** This figure shows the music for the Lazuli Bunting in the upper plot and the actual birdsong in the lower plot. The two signals exhibit a comparable time-amplitude profile, which is intriguing in of itself.



**Figure 5.** This figure shows the spectrograms for one of the sections of the Cardinal. The spectrogram of the music (above) and of the birdsong (below) highlight similar frequency content in both signals. The musical notes try to transcribe the rise and fall in the pitch of the birdsong.

### 3.2 Cauchy-Schwarz Inequality

The Cauchy-Schwarz inequality is perhaps the most important inequality in mathematics; it has widespread uses especially in functional analysis and statistics. The proof for this inequality is famous for its ingenuity. The progression of the proof is not overly strenuous, however the starting point for the proof is rather clever. Since the Cauchy-Schwarz inequality is

the main theorem upon which the correlation analysis in this paper rests on, it is useful to prove this inequality. The following is the statement and proof of the Cauchy-Schwarz inequality.

**Theorem 2** (Cauchy-Schwartz). *Let  $V$  be a vector space with inner product  $\langle \cdot, \cdot \rangle$ , and corresponding norm  $\|\cdot\|$ . Then for all  $x, y \in V$  we have*

$$|\langle x, y \rangle| \leq \|x\| \|y\| \tag{5}$$

with equality if and only if  $y = cx$  for some scalar  $c$ .

*Proof.* In the trivial case in which  $x$  or  $y$  are zero:

$$|\langle x, y \rangle| = \|x\| \|y\| = 0$$

Thus, the inequality holds in the trivial case.

Let  $x$  and  $y$  be non-zero vectors. Choose an arbitrary vector  $p(t)$  that is a function of a scalar variable,  $t$ .

$$p(t) = tx + y$$

$$\|p(t)\| = \|tx + y\| \geq 0$$

So

$$\begin{aligned} 0 \leq \|p(t)\|^2 &= \|tx + y\|^2 \\ &= \langle tx + y, tx + y \rangle \\ &= t^2 \langle x, x \rangle + 2t \langle x, y \rangle + \langle y, y \rangle \\ &= t^2 \|x\|^2 + 2t \langle x, y \rangle + \|y\|^2 \end{aligned}$$

To make the substitutions simple, let

$$\begin{aligned} a &= \|x\|^2 \\ b &= 2\langle x, y \rangle \\ c &= \|y\|^2 \end{aligned}$$

Then we have

$$\begin{aligned} 0 \leq t^2 \|x\|^2 + 2t \langle x, y \rangle + \|y\|^2 \\ = at^2 + bt + c \geq 0 \end{aligned}$$

Since the above quadratic in variable  $t$  is always greater than or equal to zero, the discriminant is negative so we have

$$b^2 - 4ac \leq 0,$$

from which it follows that

$$4\langle x, y \rangle^2 \leq 4\|x\|^2 \|y\|^2,$$

and thus

$$|\langle x, y \rangle| \leq \|x\| \|y\|.$$

Now, we need only show that equality holds if and only if  $x = cy$  where  $c$  is a some scalar. Let  $x = cy$ , then the above analysis results in

$$|\langle cy, y \rangle| \leq \|cy\| \|y\|$$

which simplifies to

$$\begin{aligned} |c| |\langle y, y \rangle| &\leq |c| \|y\|^2 \\ |c| \|y\|^2 &= |c| \|y\|^2 \end{aligned}$$

Thus, if  $x = cy$  then there is equality.

Now assume an arbitrary case with equality,

$$|\langle x, y \rangle| = \|x\| \|y\|$$

Then, from the proof:

$$4\langle x, y \rangle^2 = 4\|x\|^2 \|y\|^2$$

and with  $a, b$ , and  $c$  as previously defined we have,

$$b^2 - 4ac = 0$$

Then, by equation ??, only one real root since the discriminant is zero. Thus, the following is zero with  $t_0$  being the real root.

$$\langle t_0x + y, t_0x + y \rangle = 0$$

From which, it follows

$$\|t_0x + y\| = 0.$$

If the norm of a vector is zero, then the vector is zero.

$$t_0x + y = 0$$

This can be simplified with the value of  $t_0$  found from the quadratic mentioned above

$$x = \frac{\|x\|^2}{\langle x, y \rangle} x$$

And thus

$$x = cy$$

The vector  $x$  has to be colinear to  $y$  anytime there is equality in the Cauchy-Schwarz Theorem. This is another of saying equality if and only if  $x = cy$  for some scalar  $c$ , can there be equality. Thus the Cauchy-Schwarz inequality theorem has been proven including its condition of equality.  $\square$

### 3.3 Similarity Measures

The uncentered correlation between two vectors is based on the Cauchy-Schwarz inequality. Through simple rules of trigonometry, this correlation can also be rewritten in another form which shows the “angle” between the vectors. The uncentered correlation is derived as follows from the Cauchy-Schwarz inequality 5: If  $x$  and  $y$  are vectors in a valid vector space  $V$ ,

$$\begin{aligned} |\langle x, y \rangle| &\leq \|x\| \|y\| \\ \frac{|\langle x, y \rangle|}{\|x\| \|y\|} &\leq 1 \\ -1 &\leq \frac{\langle x, y \rangle}{\|x\| \|y\|} \leq 1 \end{aligned}$$

The above formula shows that the uncentered correlation,  $\frac{\langle x, y \rangle}{\|x\| \|y\|}$ , will always be between -1 and 1. The lower boundary represents the scenario in which the two vectors being compared are anti-parallel, and the upper boundary accounts for when the vectors are co-linear. The definition for uncentered correlation described above can be represented differently-as the angle between the two vectors.

$$\begin{aligned} \cos \theta &= \frac{\langle x, y \rangle}{\|x\| \|y\|} \\ \theta &= \arccos \left( \frac{\langle x, y \rangle}{\|x\| \|y\|} \right) \end{aligned}$$

It can be useful the view uncentered correlation between two vectors as the angle between them, as that gives one an idea to how far apart they are from each other. In other words a high uncentered correlation between two vectors suggests that they are very similar geometrically. Uncentered correlation was the first mean by which contours of birdsong and music were compared in this research project.

Centered correlation sometimes referred to as Pearson’s correlation coefficient is more of a statistical measure of similarity. It is also based on using the Cauchy-Schwarz inequality and therefore always between -1 and 1. However the main way in which it differs from uncentered correlation is that it centers both vectors to a mean of zero before correlating them. The Centered correlation between  $f$  and  $g$  is:

$$\rho_c = \frac{\text{covariance}(f, g)}{\sigma_f * \sigma_g} = \frac{\langle f - \mu_f, g - \mu_g \rangle}{\|f - \mu_f\| \|g - \mu_g\|} \quad (6)$$

The Cauchy-Schwarz theorem also holds here so we can put a bound on  $\rho_c$ .

$$\rho_c \leq 1,$$

with equality if and only if  $g(t) - \mu_g = c(f(t) - \mu_f)$  for some scalar  $c$ .

Thus, another way to think about centered correlation is that  $\rho_c$  measures essentially how well  $g$  can be approximated as a linear function of  $f$ ; at  $\rho_c = \pm 1$  there is an exact linear relationship between  $f$  and  $g$  and when  $\rho_c = 0$ ,  $f$  and  $g$  have no linear relationship. CSIM correlation is a measure of correlation between two musical contours, (which in this context are the same as vectors). It is not related to uncentered correlation or centered correlation, but rather it is based on how rises and falls in two contours compare to each other. Laparde and Marvin in their paper “Relating Musical Contours: Extensions of a Theory for Contour” explain how retention and recognition of music often depends on an invariant melodic contour, even if the “size of the interval between successive pitches may be altered” [1]. Although Laparde and Marvin cite several similarity measures in their paper, the one we found to be most useful for our research was CSIM. The algorithm by which CSIM works is as follows:

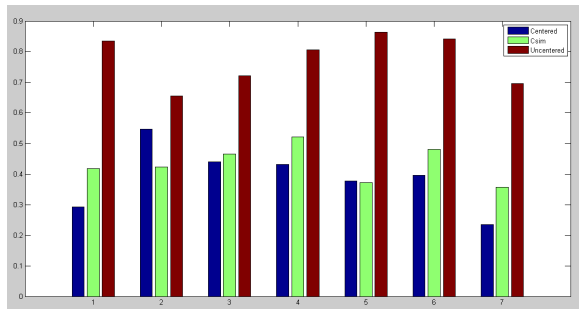
Let  $x$  and  $y$  be two vectors/contours.  $\text{CSIM}(x, y)$  returns a number between 0 and 1 that signifies the similarity between  $x$  and  $y$ ; 1 being the contours being exactly the same. Each contour is converted to COM-matrix, which is a matrix of dimensions the length of the contour. Each unit of a COM-matrix for a contour  $c$  is the corresponding row-th value of the  $c$  subtracted from the column-th value; thus a COM-matrix is just a matrix filled positive and negative numbers with the diagonal being all zero. Just the signs of the numbers in a COM-matrices are used in calculating the CSIM correlations between two contours. The CSIM correlation counts the number of corresponding signs in the

COM-matrices of the contours and then divides that number by the maximum number of similarities which is the size of the matrix.

## 4 Results

### 4.1 Fallon Pairings

The correlation values for the birdsong and the music pairings were encouraging. Figure 6 illustrates the three different correlation values calculated for each pairing.



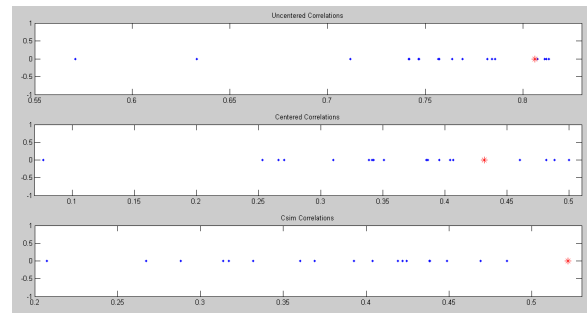
**Figure 6.** This figure shows the correlations for each of the seven pairings. The three different bars represent the three different types of correlations used. 1:Lazuli Bunting, 2:Prairie Chicken, 3:Wood Thrush, 4:Baltimore Oriole, 5:Cardinal(1), 6:Cardinal(2), 7:Cardinal(3)

As the bar graphs shows the uncentered correlations showed the highest similarities between the birdsong and the music. The CSIM correlation generally yielded higher results than the centered correlation. Some pairings had higher correlations than others.

### 4.2 Statistical Significance

The correlation data derived from Dr. Fallon's pairings of birdsong and music needs to be measured for its statistical significance. Statistical significance is very important, because it serves as a test for authenticating the validity of the results in an experiment. It assesses whether the findings actually reflect a pattern or whether they are just the result of pure chance. Because of time constraints, this research project was able to address the question of statistical significance for only one of the seven pairings analyzed in this paper. The correlation data for the Baltimore Oriole birdsong and its corresponding transcription in Messiaen's work served as the experimental data in analyzing statistical significance. Twenty randomly chosen sections of music, (each of which didn't coincide with the Baltimore Oriole section) from *Oiseaux Exotiques* were correlated against the birdsong for the Baltimore Oriole. These twenty correlations served as the control data. In this manner, the actual, experimental Baltimore Oriole pairing was measured against 20 control

pairings to highlight its statistical significance. Figure 7 helps to show this.



**Figure 7.** This figure illustrates the statistical significance for the Baltimore Oriole pairing. It shows 21 correlation values for each of the three correlation tests focused on in this paper: Uncentered correlation, Centered correlation, CSIM correlation. The correlation value denoted by an asterisk represents the actual correlation for the Baltimore Oriole pairing.

The results of the statistical significance calculated for the Baltimore Oriole were very intriguing because they turned out to be not as high as expected but nevertheless they were on the threshold of a statistical significance. Such results suggest the need for further testing to ascertain the remaining research results of this project.

## 5 Discussion/Conclusion

The statistical significance for the Baltimore Oriole pairing was not as high we would have liked, but it was also not completely trivial. Looking back, it seems like there are several factors that could have made the statistical significance for this research project better. For one, more than just 20 random correlation values representing the control would have given more accurate and possible better results. A bootstrapping analysis is a method by which we could have used different controls to see if similar statistical significance is achieved. Since the music piece for the Baltimore Oriole was the smallest of all the other music pieces for the birds, it might have been easier to find other sections of music randomly that seemed to relate to the Baltimore Oriole section. If a longer piece like the Prairie Chicken had been chosen, it would have been harder to find sections of music that resembled it. As has been mentioned above, one the reasons only the Baltimore Oriole was chosen for statistical significance was because of constraints of time near the end of this project. Another factor at play here was the mere fact that we chose our control pairings from a Messiaen piece *Oiseaux Exotiques* that focused entirely on birdsongs, which means that the randomly chosen control measures could have made the statistical significance measurements much more stringent than necessary. We conjecture that had the control

been set to another Messiaen piece that didn't deal with birdsongs as extensively, such as *Quartet for the End of Time*, the statistical significance would have been higher. The main goal of this research project has been to find objective and quantitative values of comparison between the natural birdsongs, that Olivier Messiaen so admired and was openly known for using in the compositions of his music, and his music. It is already well-known and accepted that clear similarities exist between the music and the birdsong and hearing the music or viewing it's frequency content in comparison to the birdsongs certainly makes this clear also. However, approaching the same goal from a mathematical standpoint makes finding comparisons a much more rigorous exercise. The correlation values that were derived for the seven pairing of birdsong and music overall seemed high enough to suggest that Messiaen was definitely inspired by the birdsongs he so admired. An important distinction that should be noted is that Messiaen was an artist; he was not looking to copy birdsongs into his music exactly, but rather he artistically portrayed them in his work.

## 6 Acknowledgements

In no way would this research project would have been possible without the constant help and oversight of Professor Pendergrass. Also Ms. Tammy Bishop, at the Lab of Ornithology, was quite helpful when we were trying to find recordings of birdsong needed for the research.

## References

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- [2] Robert Fallon, *The Record of Realism in Messiaen's Bird Style*. "OLIVIER MESSIAEN: Music, Art and Literature", Christopher Dingle and Nigel Simeone, Ashgate, 2007.